Theorem Proving of Enactment Model Composites with Appiah-Kleene Axiomatic Expressions (Eel Proof)

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THEOREM PROVING OF ENACTMENT MODEL COMPOSITES WITH APPIAH-KLEENE AXIOMATIC EXPRESSIONS.

(EEL Proof)

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Extended Abstract[+]. This research is on axiomatical proofing using Appiah-Kleene Axiom expressions. Nine propositional formulas from enactment logic are model proved in model terms baased on axiomatic schema means. This will result in producing the model composites proposed from the axiom scheme.

Keywords. model, composites, enactment, logic, model checking, scheme, axiom.

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1 INTRODUCTION

This research is based on axiomatical computations of Appiah-Kleene[4] that gives axiomatic expressions based on formulas of enactment models[1]. Kleene give the following axiom schemas:

(1a) $\alpha \to (\beta \to \alpha)$.

(1b) $(\alpha \to \beta) \to (\alpha \to (\beta \to \gamma) \to (\alpha \to \gamma))$.

(2a) $\alpha \to (\beta \to (\alpha \wedge \beta))$,

(2b) $(\alpha \to \beta) \to \alpha$.

(2c) $(\alpha \wedge \beta) \to \beta$.

(3a) $\alpha \to (\alpha \vee \beta)$,

(3b) $\beta \to (\alpha \vee \beta)$,

(3c) $(\alpha \to \gamma) \to ((\beta \to \gamma) \to ((\alpha \vee \beta) \to \gamma))$.

(4a) $(\alpha \to \beta) \to ((\alpha \to \neg \beta) \to \neg \alpha)$,

(4b) $(\neg\neg \alpha) \to \alpha$.

Results of Work[4]: *Appiah-Kleene Expressions*

Let $Enact_E$ to be represented by E_E .

Let $Enact_L$ to be represented by E_L .

Axiom (1a):

$(1) E_E \to (E_L \to E_E)$,
$(2) a_i \to (a \to a_i)$,
$(3) a_j \to (l \to a_j)$,
$(4) rank_i \to (t \to rank_i)$.

Axiom (2a)

$(1) E_E \to (E_L \to E_E \wedge E_L)$,
$(2) a_i \to (a \to a_i \wedge a)$,
$(3) a_j \to (l \to a_j \wedge l)$,
$(4) rank_i \to (t \to rank_i \wedge t)$.

Axiom (1b):

$(1) (E_E \to E_L) \to ((E_E \to (E_L \to a_i)) \to (E_E \to a_i))$,
$(2) (a_i \to a) \to ((a_i \to (a \to a_i)) \to (a \to a_i))$,
$(3) (a_j \to l) \to ((a_j \to (l \to a_j)) \to (l \to a_j))$,
$(4) (rank_i \to t) \to ((rank_i \to (t \to rank_i)) \to (t \to rank_i))$.

Axiom (2b):

$(1) (E_E \wedge E_L) \to E_E$,
$(2) (a_i \wedge a) \to a_i$,
$(3) (a_j \wedge l) \to a_j$,
$(4) (rank_i \wedge t) \to rank_{i.}$

Axiom (2c):

$(1) (E_E \wedge E_L) \to E_L$,
$(2) (a_i \wedge a_i) \to a_i$,
$(3) (a_j \wedge l) \to l$,
$(4) (rank_i \wedge t) \to t.$

Axiom (3a):

$(1) E_E \to (E_E \vee E_L)$,
$(2) a_i \to (a_i \vee a)$,
$(3) a_j \to (a_j \vee l)$,
$(4) rank_i \to (rank_i \vee t)$.

Axiom (3b):

$(1) E_L \to (E_E \vee E_L)$,
$(2) a \to (a_i \vee a)$,
$(3) l \to (a_j \vee l)$,
$(4) t \to (rank_i \vee t)$.

Axiom (3c):

$(1) (E_E \to a_i) \to ((E_L \to a_i) \to ((E_E \vee E_L) \to a_i))$,
$(2) (a_i \to a) \to ((a_i \to a) \to ((a_i \vee a) \to a_i))$,
$(3) (a_j \to l) \to ((a_j \to l) \to ((a_j \vee l) \to a_j))$,
$(4) (rank_i \to t) \to ((rank_i \to t) \to ((rank_i \vee t) \to rank_i))$.

Axiom (4a):

$$(1)(E_E \rightarrow E_L) \rightarrow ((E_E \rightarrow \neg E_L) \rightarrow \neg E_E),$$
$$(2)(a_i \rightarrow a) \rightarrow ((a_i \rightarrow \neg a) \rightarrow \neg a_i),$$
$$(3)(a_j \rightarrow l) \rightarrow ((a_j \rightarrow \neg l) \rightarrow \neg a_j),$$
$$(4)(rank_i \rightarrow t) \rightarrow ((rank_i \rightarrow \neg t) \rightarrow \neg rank_i).$$

Axiom (4b):

$$(1) \neg\neg E_E \rightarrow E_E,$$
$$(2) \neg\neg E_L \rightarrow E_L,$$
$$(3) \neg\neg a_i \rightarrow a_i,$$
$$(4) \neg\neg a_j \rightarrow a_j,$$
$$(5) \neg\neg l \rightarrow l,$$
$$(6) \neg\neg t \rightarrow t,$$
$$(7) \neg\neg rank_i \rightarrow rank_{i.}$$

2 THEOREM PROVING OF MODEL COMPOSITES

The models of enactment logic are:

(1) $enact_E \rightarrow enact_L$

(2) $a_i \rightarrow a$

(3) $a_j \rightarrow l$

(4) $rank_i \rightarrow t$

(5) $enact_E \rightarrow (enact_E \rightarrow a_i)$

(6) $eanct_L \rightarrow (a_j \rightarrow l)$

(7) $a_i \rightarrow (a_j \rightarrow l)$

(8) $a_j \rightarrow (l \rightarrow rank_i)$

(9) $l \rightarrow (rank_i \rightarrow t)$

Theorems are proven by substituting a general possible axiom into other to deduce a model composite by resolution and this is called *unification*. Models (1) to (9) will be proved using Appiah-Kleene expression axioms (1a) to (4b).

(1) $enact_E \rightarrow enact_L$: *Enact Model*

(1) is expressed by Appiah-Kleene Axioms (3b and 3a) as shown below:

Axiom (3b-clause 1) is substituted into Axiom (3a-clause 1) at the right-hand side to give a proof theorem.

Axiom (3b-clause 1): $E_L \rightarrow (E_E \vee E_L)$

Axiom (3a-clause 1): $E_E \rightarrow (E_E \vee E_L)$

$$E_E \rightarrow (E_E \vee E_L) = (E_E \vee E_L) \leftarrow E_L$$
$$E_E \rightarrow 1 \leftarrow E_L \quad : Unification$$

Proof. $E_E \rightarrow (1 \leftarrow E_L) : Resolution$

$$E_E \rightarrow ((1 \leftarrow E_L) = E_L)$$
$$E_E \rightarrow E_{L.}$$

Model theorem of enactment is proved by unification and resolution. Unification because (3b-c1) had its right-hand or-clause equalizing (3a-c1) left-hand or-clause. Resolution because (3b-c1) and (3a-c1) had both E_E and E_L approaching 1 from both hands. In simple life, one does model or sculpt a thing with both hands. Let get our hands into making something for simple unity and achieving a resolution. Chang&Lee[5] did find a unification algorithm for theorem proving for predicate logic.

Theorem 1: Enact model is a provable theorem.

(2) $a_i \rightarrow a$: *Interest-Action Model*

(1) is expressed by Appiah-Kleene Axioms (3b and 3a) as shown below:

Axiom (3b-clause 2) is substituted into Axiom (3a-clause 2) at the right-hand side to give a proof theorem.

Axiom (3b-clause 2): $a \rightarrow (a_i \vee a)$

Axiom (3a-clause 2): $a_i \rightarrow (a_i \vee a)$

$$a_i \rightarrow (a_i \vee a) = (a_i \vee a) \leftarrow a$$
$$a_i \rightarrow 1 \leftarrow a \quad : Unification$$

Proof. $a_i \rightarrow (1 \leftarrow a) : Resolution$

$$a_i \rightarrow ((1 \leftarrow a) = a)$$
$$a_i \rightarrow a.$$

Theorem 2: Interest-Action model is a provable theorem in Appiah-Kleene Axioms.

(3) $a_j \rightarrow l$: *Interest-Location Model*

(1) is expressed by Appiah-Kleene Axioms (3b and 3a) as shown below:

Axiom (3b-clause 3) is substituted into Axiom (3a-clause 3) at the right-hand side to give a proof theorem.

Axiom (3b-clause 3): $l \rightarrow (a_j \vee l)$

Axiom (3a-clause 3): $a_j \rightarrow (a_j \vee l)$

$$a_j \rightarrow (a_j \vee l) = (a_j \vee l) \leftarrow l$$
$$a_j \rightarrow 1 \leftarrow l \quad : Unification$$

Proof. $a_j \rightarrow (1 \leftarrow l) : Resolution$

$$a_j \rightarrow ((1 \leftarrow l) = l)$$
$$a_j \rightarrow l.$$

Theorem 3: Interest-Location model is a simple provable theorem in Appiah-Kleene Axioms.

(4) $rank_i \rightarrow t$: *Rank-Time Model*

(1) is expressed by Appiah-Kleene Axioms (3b and 3a) as shown below:

Axiom (3b-clause 4) is substituted into Axiom (3a-clause 4) at the right-hand side to give a proof theorem.

Axiom (3b-clause 4): $t \rightarrow (rank_i \vee t)$

Axiom (3a-clause 4): $rank_i \rightarrow (rank_i \vee t)$

$$rank_i \rightarrow (rank_i \vee t) = (rank_i \vee t) \leftarrow t$$
$$rank_i \rightarrow 1 \leftarrow t \quad : Unification$$

Proof. $$rank_i \rightarrow (1 \leftarrow t) : Resolution$$
$$rank_i \rightarrow ((1 \leftarrow t) = t)$$
$$rank_i \rightarrow t.$$

Theorem 4: Interest-Location model is a provable theorem in Appiah-Kleene Axioms (3b-clause 4 and 3a-clause 4).

3 CONCLUSION

This research work concludes on axiomatic expressions constructed from Appiah-Kleene axiom expressions. In here, expressive enactment logic is used to model proven the composites of enactment logic. Theorem (1) to (4) of model composites of enactment are proven by only a simple unification from Appiah-Kleene Axioms (3a) and (3b).

Compliance with Ethical Standards

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Conflict of Interest:

Author, Dr. Frank Appiah declares that he has no conflict of interest .

REFERENCES

1. Appiah F. (2020). Semantic Computation of Propositional Model Composites in Enactment Logic, KCL Art & Science Research Office, Waterloo, England, United Kingdom.
2. Appiah F. (2009/10), RuleML for Policy Exchange in Agent Commerce, King's College London, Msc Dissertation.
3. Richard Stark W.(1990), Lisp, Lore, Logic, Springer Verlag, New York.
4. Appiah F. (2020), Axiomatical Computation of Model Composites of Enactment Logic. KCL School of Engineering, England, United Kingdom.
5. Chang, Chin-Lang and Richard Char-Tung Lee(1973). Symbolic Logic and Mechanical Theorem Proving, Academic Press, New York.