Solving Knapsack Problem with Interval Type-2 Triangular Fuzzy Numbers

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Abstract In this work, a mean value footprint of uncertainty(MVFOU) method has been proposed for solving Fuzzy Knapsack Problem with Interval Type-2 Triangular Fuzzy numbers as its coefficients. The Knapsack problem is a combinatorial optimization problem with the goal of finding, in a set of items of given prices and weights, the subset of items with the maximum total price, subject to a total weight constraint. In the Knapsack problem, the given items have two attributes at a minimum - an item's value which affects its importance, and an item's weight, which is its limitation aspect. There exist some cases in which the precise value of weight or(and) prices is(are) not specified, but rather a range for these is given. Complexity arises when the possibility of falling of these values within a specified range is also not fixed. In such cases, instead of using real numbers, we rather use fuzzy numbers or more specifically an interval type-2 fuzzy numbers as the coefficients for the values and/or weight of the items. In the first stage of solving a fuzzy knapsack problem with interval type-2 triangular fuzzy numbers, the problem is converted into three different crisp knapsack problems which are then solved optimally by using dynamic programming methods. Then in the second stage, with the help of PGMIR method, feasible solution(s) are selected. The feasible solution for which the profit earned is maximum is known as optimal solution of the problem. In order to explain the methodology, a numerical example has also been worked out in this paper.

Keywords Fuzzy Knapsack Problem · Interval Type 2 Triangular Fuzzy Numbers · Mean Value and Footprint of uncertainty method · Dynamic Programming method · Parametric Graded Mean Integration Representation method.

1 Introduction

The knapsack problem [20] is a combinatorial optimization problem in which the decision maker is given a set of items, each with a weight and a value and the objective is to determine a sub-collection of these items so that the total weight is less than or equal to a given limit, often known as capacity of knapsack and the total value is as large as possible. The problem appears very often in real-world decision making processes. One of the very early applications of knapsack algorithm is in the construction and scoring of tests [7], where the nature of questions is heterogeneous, and the test-takers have a choice as to which questions they answer. The very first works regarding Knapsack problem is dated as far back as 1897 [16]. The knapsack problem find its real world application in resource allocation [1], selection of investment and portfolios [11], finding a least wasteful way to cut raw materials, selection of assets for asset-based securitization and many more.

Various classes of algorithms based on greedy method, linear programming relaxation method, dynamic programming method, branch-and-bound method, approximation methods have been defined in the literature to solve a knapsack problem, where the data under consideration is given precisely. But, in real life situation, precision of data is not always guaranteed and the value of weights or prices of items are given imprecisely. In such situations, the knapsack problem gets extended to the fuzzy knapsack problem and in such cases, the fuzzy set theory [21] can be used to solve the problem. In [14], a fuzzy knapsack problem,

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where the weights of items are represented by triangular fuzzy numbers has been discussed. The 0-1 knapsack problem with fuzzy data is also discussed by Kasperski and Kulej in [10].

Apart from various exact algorithms, some algorithms based on meta-heuristics, hyper-heuristics and evolutionary optimization have been proposed to solve fuzzy knapsack problem [15]. In [4], refining and repairing operations have been introduced and an improved genetic algorithm has been used to solve fuzzy knapsack problem, where profits and weights have been considered as a fuzzy numbers. In [3], a novel ant colony optimization algorithm have been used to solve binary knapsack problem. In this work, trapezoidal fuzzy numbers have been used to represent profits and weights of items. In [5], a parametric programming approach has been used to solve fuzzy knapsack problem. In [17], a hyper-heuristic method has been defined for solving knapsack problem through fuzzy logic. The fuzzy knapsack problem with imprecise weight coefficients has been solved by using genetic algorithm in [13]. In [19], a dynamic programming approach using multi-stage decision making method has been given for solving fuzzy knapsack problem. A dynamic programming based method has been used for solving fuzzy knapsack problem by introducing a possibility index which determines the possibility for choosing an item with fuzzy weight to be included in a knapsack whose capacity is given by a crisp number in [2]. In [18], bi-objective fuzzy knapsack problem has been solved by using Dynamic Programming algorithm.

In this paper, a defuzzification method based on mean value footprint of uncertainty method is proposed for solving fuzzy knapsack problems where the weights and profits associated with the items are given by interval type-2 triangular fuzzy numbers(IT2TFNs). This paper is organized as follows: In Section 2, some definitions and concepts regarding fuzzy set theory and dynamic programming algorithm to solve knapsack problem have been reviewed. In Section 3, a fuzzy knapsack problem with IT2TFNs as coefficients representing weights, profits and knapsack capacity has been formulated and its solution methodology has been discussed. The algorithm for solving the problem has been proposed in Section 4. A numerical example has been presented in Section 5. The last section comprises of the concluding remarks.

2 Preliminaries: Concepts and Definitions

2.1 Definitions:

Definition 1 Fuzzy Set [21]: If X is a universe of discourse and x is a particular element of X, then a fuzzy set $\tilde{A}$ defined on X can be written as collection of ordered pairs.

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\} \tag{1}$$

where $\mu_{\tilde{A}}(x) : X \to [0,1]$ is known as membership function and it entails the degree of membership of elements of X in fuzzy set $\tilde{A}$.

Definition 2 Fuzzy Number [12]: A fuzzy set $\tilde{A}$ on $\mathcal{R}$ is said to be a fuzzy number, if the following three properties are satisfied:

1. Fuzzy set $\tilde{A}$ must be normal,
   i.e. $\exists\, x$ such that $sup\, \mu_{\tilde{A}}(x) = 1$, where sup stands for supremum.
2. The support of fuzzy set, i.e. set of all the elements with non zero degree of membership, must be bounded.
3. α level set i.e. set of all the elements with membership degree greater than α, must be a closed interval for $\alpha \in$ [0,1].

Definition 3 Triangular Fuzzy Number [21]: A fuzzy number $\tilde{A}$ is said to be a triangular fuzzy number if the graph of its membership function is triangular in shape. The membership function of a triangular fuzzy number is defined by equation (2) and the membership function of such a triangular fuzzy number is represented by Figure 1.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & if\, x < a \\ \dfrac{x-a}{b-a} & if\, a \le x \le b \\ \dfrac{c-x}{c-b} & if\, b \le x \le c \\ 0 & if\, x > c \end{cases} \tag{2}$$

The triangular fuzzy number is denoted by (a, b, c).

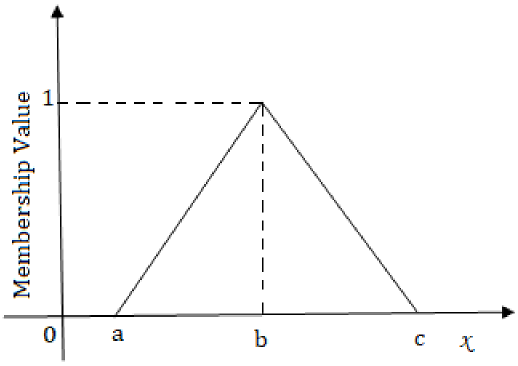

Fig. 1: A Triangular Fuzzy Number

Definition 4 Interval-valued fuzzy Set [12]: If the membership function is defined such that it assigns to each element of universal set X a closed interval [a,b]⊆[0,1] then the fuzzy set defined by such membership function is called interval-valued fuzzy set. The membership function of this type of fuzzy set is defined as:

$$\mu_{\tilde{A}} : X \rightarrow \varepsilon([0,1]) \tag{3}$$

where $\varepsilon([0,1])$ denotes the family of all closed intervals of real numbers in [0,1].

Definition 5 Type-2 Fuzzy Set [12]: Interval-valued fuzzy set can further be generalized by allowing the intervals [a,b] to be fuzzy. Such type of fuzzy sets are called as Type-2 Fuzzy Sets. Their membership function is defined as:

$$\mu_{\tilde{A}} : X \rightarrow \mathbb{F}([0,1]) \tag{4}$$

where $\mathbb{F}([0,1])$ denotes the set of all ordinary fuzzy sets that can be defined on interval [0,1] called as Fuzzy Power Set of [0,1] and $\tilde{A}$ is defined as:

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \tag{5}$$

Definition 6 Interval Type-2 Fuzzy Set [12]: In the definition of type-2 fuzzy set if $\mu_{\tilde{A}}(x,u)$=1 $\forall x$ and $\forall u$ then it is called interval type-2 fuzzy set, i.e.,

$$\tilde{A} = \{((x,u), 1) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\}$$

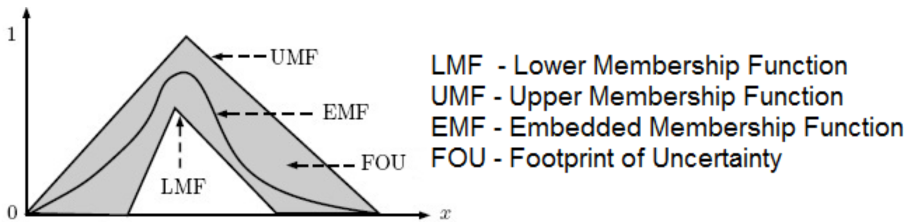

Fig. 2: Interval Type-2 Fuzzy Set

Definition 7 Interval Type-2 Fuzzy Number [9]: An interval type-2 fuzzy set $\tilde{A} = [\underline{A}, \overline{A}]$ is said to be Interval Type-2 Fuzzy Number if its LMF(lower membership function), $\underline{A}$, and UMF(upper membership function), $\overline{A}$, are type-1 fuzzy numbers.

Definition 8 Interval type 2 Triangular Fuzzy Number [9]: An IT2TFN, $\tilde{A}$ is defined on the interval $[\overline{a}, \overline{c}]$, its lower membership function and upper membership function, takes the value equal to $\underline{h} \in [0,1]$ at $\underline{b}$ and $\overline{h} \in [0,1]$ at $\overline{b}$, respectively where

$$\overline{\check{a}} \leq \underline{\check{a}} \leq \underline{a} \leq \overline{a} \leq \underline{\hat{a}} \leq \overline{\hat{a}}$$

Thus, IT2TFN $\tilde{A}$ is notated as

$$\tilde{A} = (\underline{A}, \overline{A}) = \left((\underline{\check{a}}, \underline{a}, \underline{\hat{a}}), (\overline{\check{a}}, \overline{a}, \overline{\hat{a}})\right)$$

An IT2TFN is represented by Figure 3.

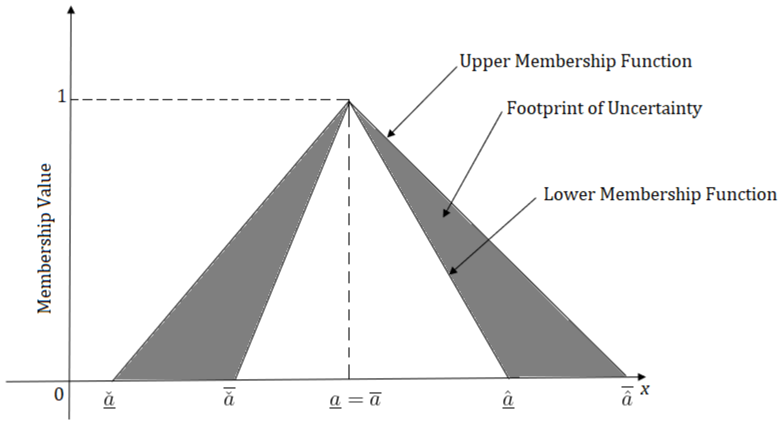

Fig. 3: Interval Type-2 Triangular Fuzzy Number

Definition 9 Arithmetic Operations on Interval Type-2 Triangular Fuzzy Numbers [9]**:** If $\tilde{A} = \left((\underline{\check{a}}, \underline{a}, \underline{\check{a}}), (\overline{\hat{a}}, \overline{a}, \overline{\hat{a}})\right)$ and $\tilde{B} = \left((\underline{\check{b}}, \underline{b}, \underline{\check{b}}), \left(\overline{\hat{b}}, \overline{b}, \overline{\hat{b}}\right)\right)$ are two IT2TFNS, then

1. Addition:

$$\tilde{A} \oplus \tilde{B} = \left(\left(\underline{\check{a}} + \underline{\check{b}}, \underline{a} + \underline{b}, \underline{\hat{a}} + \underline{\hat{b}}\right), \left(\overline{\check{a}} + \ \overline{\check{b}}, \overline{a} + \overline{b}, \overline{\hat{a}} + \overline{\hat{b}}\right)\right)$$

2. Subtraction:

$$\tilde{A} \ominus \tilde{B} = \left(\left(\underline{\check{a}} - \underline{\hat{b}}, \underline{a} - \underline{b}, \underline{\hat{a}} - \underline{\check{b}}\right), \left(\overline{\check{a}} - \ \overline{\hat{b}}, \overline{a} - \overline{b}, \overline{\hat{a}} - \overline{\check{b}}\right)\right)$$

3. Scalar Multiplication:

$$k \odot \tilde{A} = \begin{cases} \left((k\underline{\check{a}}, k\underline{a}, k\underline{\check{a}}), (k\overline{\hat{a}}, k\overline{a}, k\overline{\hat{a}})\right) & k \geq 0 \\ \left((k\underline{\hat{a}}, k\underline{a}, k\underline{\check{a}}), (k\overline{\hat{a}}, \overline{a}, \overline{\check{a}})\right) & k < 0 \end{cases}$$

Definition 10 Parametric Graded Mean Integration Representation Method [8]**:** Let $L(x)$ and $R(x)$ be the left and right fuzzy functions of a fuzzy number $\tilde{A}$, then the PGMIR representation of $\tilde{A}$ is given by

$$P_{\tilde{A}} = \frac{\int_0^1 \frac{x}{2}\left(L^{-1}(x) + R^{-1}(x)\right) dx}{\int_0^1 \frac{x}{2} dx} \tag{6}$$

where $0 \leq h \leq 1$ is height of $\tilde{A}$.
For the IT2TFN, $\tilde{A} = \left((\underline{\check{a}}, \underline{a}, \underline{\check{a}}), (\overline{\hat{a}}, \overline{a}, \overline{\hat{a}})\right)$, the PGMIR expression is given by:

$$P_{\tilde{A}} = \frac{1}{12}\left(\underline{\check{a}} + \underline{\hat{a}} + \overline{\check{a}} + \overline{\hat{a}}\right) + \frac{1}{3}\left(\overline{a} + \underline{a}\right) \tag{7}$$

2.2 **Concepts:**

Dynamic Programming method to solve Unbounded Knapsack Problem [6]:

In unbounded knapsack problem, a knapsack of certain capacity is given and 'n' items with certain profits and weights are given. The objective is to fill the knapsack in such a way that profit is maximized while following the weight constraint. In case of unbounded knapsack problem, multiple copies of a single item are allowed. Dynamic Programming methods are used to solve large problems by storing the solutions of slightly smaller problems, known as sub-problems. The solution of these sub-problems is usually stored in a table, so that these sub-problems are never calculated more than one time. Unbounded Knapsack problem can be solved by considering knapsacks of lesser capacities as sub-problems, storing best profit for each capacity. The optimal solution for a knapsack of capacity C can then be found by using the solution for knapsacks with capacities lesser than C.

A vector p is calculated, whose elements $p[c]$ stores the best profit possible for capacity c, $0 < c \leq C$. The dynamic programming programming algorithm for the unbounded knapsack problem is shown in Algorithm 1. In the first line, $p[0]$ is set to 0 since the profit that can happen when the knapsack capacity is filled with 0 weight is 0. Then for every capacity , c from 1 to C, the item type that fits the current knapsack and yields largest profit is used. After p is calculated for every c, the optimal solution can be obtained on $p[C]$.

Algorithm 1 A Dynamic Programming algorithm to solve unbounded knapsack problem.

Input: The set of items with its weights and prices and capacity of knapsack.
Output: The profit obtained when capacity of knapsack is C.

```
1: Start
2: p[0] = 0;
3: for all c := 1 to C do
4:    p[c] := p[c − 1];
5:    for all item type i ∈ N do
6:       if w_i ≤ c then
7:          p[c] = maxp[c], p[c − w_i] + p_i
8:       end if
9:    end for
10: end for
11: return p[C]
```

3 Problem Formulation and Model Development

A mathematical model to solve knapsack problem with interval type-2 fuzzy numbers is:-

$$\begin{aligned} &\text{Maximize} \quad \tilde{z} = \textstyle\sum_{i=1}^{n} \tilde{p_i} x_i \\ &\text{such that} \\ &\quad \textstyle\sum \tilde{w_i} x_i \leq \tilde{W} \\ &\quad x_i \in \{0, 1, 2, \cdots\} \qquad i = 1, 2, \cdots, n \end{aligned} \tag{8}$$

where $\tilde{p_i}'s$ and $\tilde{w_i}'s$ are prices and weights of i^{th} item respectively and $\tilde{w}$ is the capacity of the given knapsack. Here, $\tilde{p_i}'s$ and $\tilde{w_i}'s$ and $\tilde{w}$ are interval type-2 fuzzy numbers.

In this work, we convert the given fuzzy knapsack problem into deterministic problem using the following models:- First, we convert the fuzzy model given by eq. (8) into three deterministic models namely Mean Value model, Lower Mid Value model and Upper Mid Value model given by eq. (9), eq. (10) and eq. (11) respectively.

Mean Value Model:

$$\begin{aligned} &\text{Maximize} \quad \overline{z} = \textstyle\sum_{i=1}^{n} \overline{p_i} x_i \\ &\text{such that} \\ &\quad \textstyle\sum \overline{w_i} x_i \leq \overline{W} \\ &\quad x_i \in \{0, 1, 2, \cdots\} \qquad i = 1, 2, \cdots, n \end{aligned} \tag{9}$$

Lower Mid Value Model:

$$
\begin{aligned}
&\text{Maximize} \quad \hat{z} = \sum_{i=1}^{n} \frac{2\overline{p_i} + \overline{\check{p}_i} + \underline{\check{p}_i}}{4} x_i \\
&\text{such that} \\
&\sum_{i=1}^{n} \left[\frac{2\overline{w_i} + \overline{\check{w}_i} + \underline{\check{w}_i}}{4}\right] x_i \leq \left[\frac{2\overline{W} + \overline{\check{W}} + \underline{\check{W}}}{4}\right] \\
&x_i \in \{0, 1, 2, \cdots\} \qquad i = 1, 2, \cdots, n
\end{aligned}
\tag{10}
$$

Upper Mid Value Model:

$$
\begin{aligned}
&\text{Maximize} \quad \hat{z} = \sum_{i=1}^{n} \frac{2\overline{p_i} + \overline{\hat{p}_i} + \underline{\hat{p}_i}}{4} x_i \\
&\text{such that} \\
&\sum_{i=1}^{n} \left[\frac{2\overline{w_i} + \overline{\hat{w}_i} + \underline{\hat{w}_i}}{4}\right] x_i \leq \left[\frac{2\overline{W} + \overline{\hat{W}} + \underline{\hat{W}}}{4}\right] \\
&x_i \in \{0, 1, 2, \cdots\} \qquad i = 1, 2, \cdots, n
\end{aligned}
\tag{11}
$$

Solve eq.(9), eq. (10) and eq. (11) using dynamic programming model and note down the solution. Among the three solutions obtained, find out the solution for which the sum of PGMIR of weights of selected items is less than or equal to the PGMIR of the knapsack quantity. These solutions are then termed as feasible solutions of the given knapsack problem. To find the optimal solution, we calculate the PGMIR of the prices and the sum of these PGMIR prices for each feasible solution. A feasible solution for which the sum of PGMIR of prices is largest is known as the optimal solution of the given knapsack problem.

4 Algorithm

In this section, an algorithm to solve knapsack problem with IT2TFNs is presented. The solution procedure developed in Section 3 can be summarized as:

Step 1: First convert the given knapsack problem with IT2TFNs in three deterministic models namely mean value model, lower mid value model and upper mid value model.

Step 2: Solve each problem using the dynamic programming method and note down the solution.

Step 3: Find out the PGMIR of weights of each item and the capacity of the given knapsack.

Step 4: Find out the solution, among the solutions obtained in step 2, for which the sum of PGMIR of weights of selected items is less than or equal to the PGMIR of the knapsack quantity.

Step 5: The solution(s) obtained in Step 4 is(are) feasible solution(s) of the given knapsack problem.

Step 6: Calculate the PGMIR of prices and find out the sum of PGMIR of prices for each feasible solution.

Step 7: The solution for which the sum of PGMIR of prices is largest is known as the optimal solution of the given knapsack problem.

5 Numerical Example

Consider the fuzzy knapsack problem with Interval Type 2 Triangular fuzzy numbers as,

$$
\begin{aligned}
&\text{Maximize} \quad \tilde{p_1} x_1 + \tilde{p_2} x_2 + \tilde{p_3} x_3 + \tilde{p_4} x_4 \\
&\text{such that} \\
&\tilde{w_1} x_1 + \tilde{w_2} x_2 + \tilde{w_3} x_3 + \tilde{w_4} x_4 \leq \tilde{W} \\
&x_1, x_2, x_3, x_4 \in \{0, 1, 2, \cdots\}
\end{aligned}
\tag{12}
$$

where

Item	Weight	Price
1	$\tilde{w}_1$=[(0.5,1,1.5),(0.25,1,1.75)]	$\tilde{p}_1$=[(0.75,1,1.25),(0.5,1,1.5)]
2	$\tilde{w}_2$=[(2.5,3,3.5),(2,3,4)]	$\tilde{p}_2$=[(4,5,6),(3,5,7)]
3	$\tilde{w}_3$=[(3,4,5),(2,4,6)]	$\tilde{p}_3$=[(6,7,8),(5,7,9)]
4	$\tilde{w}_4$=[(5,6,7),(4.5,6,7.5)]	$\tilde{p}_4$=[(9.5,11,12.5),(9,11,13)]

Capacity $\tilde{W}$=[(16,17,18),(15,17,19)]

Step 1: First we convert the given fuzzy model into three deterministic models namely mean value model, lower mid value model and upper mid value model as follows:-

Mean Value Model:

$$\begin{aligned} &\text{Maximize} \quad \overline{z} = x_1 + 5x_2 + 7x_3 + 11x_4 \\ &\text{such that} \\ &x_1 + 3x_2 + 4x_3 + 6x_4 \leq 17 \\ &x_1, x_2, x_3, x_4 \in \{0, 1, 2, \cdots, n\} \end{aligned} \tag{13}$$

Lower Mid Value Model:

$$\begin{aligned} &\text{Maximize} \quad \hat{z} = 0.8125x_1 + 4.25x_2 + 6.25x_3 + 10.125x_4 \\ &\text{such that} \\ &0.6875x_1 + 2.625x_2 + 3.25x_3 + 5.375x_4 \leq 16.25 \\ &x_1, x_2, x_3, x_4 \in \{0, 1, 2, \cdots, n\} \end{aligned} \tag{14}$$

Upper Mid Value Model:

$$\begin{aligned} &\text{Maximize} \quad \hat{z} = 1.1875x_1 + 5.75x_2 + 7.75x_3 + 11.875x_4 \\ &\text{such that} \\ &1.3125x_1 + 3.375x_2 + 4.75x_3 + 6.625x_4 \leq 17.75 \\ &x_1, x_2, x_3, x_4 \in \{0, 1, 2, \cdots, n\} \end{aligned} \tag{15}$$

Step 2: Solving model (13), we get $(x_1, x_2, x_3, x_4) = (1, 0, 1, 2)$ and $\overline{z} = 30$. Solving model (14), we get $(x_1, x_2, x_3, x_4) = (0, 0, 5, 0)$ and $\hat{z} = 31.25$ and solving model (15), we get $(x_1, x_2, x_3, x_4) = (0, 1, 0, 2)$ and $\check{z} = 29.5$

Step 3: Calculate the PGMIR for all the weights and knapsack capacity.

$$\begin{aligned} P_{\tilde{w}_1} &= \frac{1}{12}(0.5 + 1.5 + 0.25 + 1.75) + \frac{1}{3}(2) \\ &= 1 \\ P_{\tilde{w}_2} &= \frac{1}{12}(2.5 + 3.5 + 2 + 4) + \frac{1}{3}(6) \\ &= 3 \\ P_{\tilde{w}_3} &= \frac{1}{12}(3 + 5 + 2 + 6) + \frac{1}{3}(12) \\ &= 4 \\ P_{\tilde{w}_4} &= \frac{1}{12}(5 + 7 + 4.5 + 7.5) + \frac{1}{3}(12) \\ &= 6 \\ P_{\tilde{W}} &= \frac{1}{12}(16 + 18 + 15 + 19) + \frac{1}{3}(34) \\ &= 17 \end{aligned}$$

Step 4: The solutions obtained in Step 2 are now examined if the PGMIR of weights of selected items is less than the PGMIR of knapsack quantity.

The PGMIR of weights of selected items according to mean value model is given by:

$$a_1 = 1,\ a_2 = 0,\ a_3 = 1,\ a_4 = 2$$

$$\Rightarrow P_{\tilde{w}_1} + P_{\tilde{w}_3} + 2P_{\tilde{w}_4} = 1 + 4 + 12 = 17$$

The PGMIR of weights of selected items according to lower mid value model is given by:

$$a_1 = 0,\ a_2 = 0,\ a_3 = 5,\ a_4 = 0$$

$$\Rightarrow 5P_{\tilde{w}_3} = 5(4) = 20$$

The PGMIR of weights of selected items according to upper mid value model is given by:

$$a_1 = 0,\ a_2 = 1,\ a_3 = 0,\ a_4 = 2$$

$$\Rightarrow P_{\tilde{w}_2} + 2P_{\tilde{w}_4} = 3 + 2(6) = 15$$

Step 5: From this, we can conclude that the solution obtained by the mean value model and upper mid value model satisfy the weight constraint. So, both of these solutions are feasible solutions.

Step 6: Now, we calculate the PGMIR for prices and find out the prices for each feasible solution.

$P_{\tilde{p}_1} \ = \ \frac{1}{12}[0.75 + 1.25 + 0.5 + 1.5] + \frac{1}{3}[2] \ = \ 1$

$P_{\tilde{p}_2} \ = \ \frac{1}{12}[4 + 6 + 3 + 7] + \frac{1}{3}[10] \ = \ 5$

$P_{\tilde{p}_3} \ = \ \frac{1}{12}[6 + 8 + 5 + 9] + \frac{1}{3}[14] \ = \ 7$

$P_{\tilde{p}_4} \ = \ \frac{1}{12}[9.5 + 12.5 + 9 + 13] + \frac{1}{3}[22] \ = \ 11$

Now, we calculate the profit that can be made by using the solutions of mean value model and upper mid value model.

The PGMIR of prices of selected items according to mean value model is given by:

$$a_1 = 1,\ a_2 = 0,\ a_3 = 1,\ a_4 = 2$$

$$\Rightarrow P_{\tilde{p}_1} + P_{\tilde{p}_3} + 2P_{\tilde{p}_4} = 1 + 7 + 22 = 30$$

The PGMIR of prices of selected items according to upper mid value model is given by:

$$a_1 = 0,\ a_2 = 1,\ a_3 = 0,\ a_4 = 2$$

$$\Rightarrow P_{\tilde{p}_2} + 2P_{\tilde{p}_4} = 5 + 2(11) = 27$$

Step 7: Since the PGMIR of profit is larger for the solution obtained by mean value model. So, the solution for the given knapsack problem with IT2TFNs is $a_1 = 1,\ a_2 = 0,\ a_3 = 1,\ a_4 = 2$.

The final solution obtained is as follows:

$$\tilde{z} = [(25.75, 30, 34.25), (23.5, 30, 36.5)]$$

$$\tilde{w} = [(13.5, 17, 20.5), (11.25, 17, 22.75)]$$

where $\tilde{z}$ is the total price of the selected items and $\tilde{w}$ is the total weights of the selected items.

Figure 4 represents the graphical representation of the profit earned in the solution of eq. (12). Figure 5 represents the total weight of selected items obtained as optimal solution and the capacity of the given knapsack.

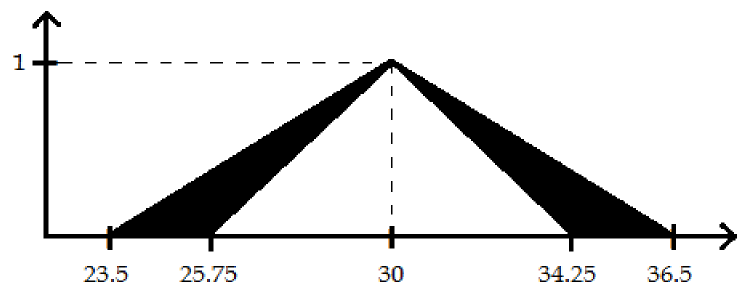

Fig. 4: Total Price of the items

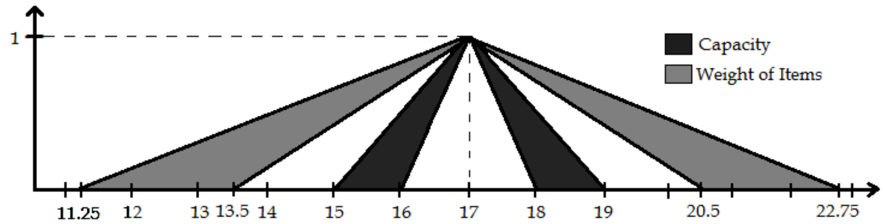

Fig. 5: Total weight of the items and capacity of the given Knapsack.

6 Conclusion

In this work, the knapsack problem in crisp environment is extended to the knapsack problem in fuzzy environment. In real world, the process of information retrieval is costly and a precise data is not always available. In such case, instead of a precise value, a range of values for the coefficients is made available and in order to handle such imprecise data, fuzzy set theory is used. In this work, the ranges of the coefficients representing item's weight and item's value is not fixed and hence are given by interval type-2 triangular fuzzy numbers. The fuzzy model of knapsack problem is defuzzified and three crisp models of the problem are formed; which are then solved by using dynamic programming method and an optimal solution to the problem is sought. In this work, a numerical example has also been solved using the proposed methodology. These results will come handy while solving a multi-objective knapsack problem in a fuzzy environment. The method explained will also help researchers to cover more realistic situations while modelling knapsack problem.

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