EasyChair Preprint

№ 11121

Logic, Mathematics, Physics: from a Loose Thread to the Close Link or What Gravity Is for Both Logic and Mathematics Rather than Only for Physics

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October 23, 2023

Logic, mathematics, physics: from a loose thread to the close link: Or what gravity is for both logic and mathematics rather than only for physics

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Abstract. Gravitation is interpreted to be an "ontomathematical" force or interaction rather than an only physical one. That approach restores Newton's original design of universal gravitation in the framework of "The Mathematical Principles of Natural Philosophy", which allows for Einstein's special and general relativity to be also reinterpreted ontomathematically. The entanglement theory of quantum gravitation is inherently involved also ontomathematically by virtue of the consideration of the qubit Hilbert space after entanglement as the Fourier counterpart of pseudo-Riemannian space. Gravitation can be also interpreted as purely mathematical or logical "force" or "interaction" as a corollary from its ontomathematical (rather than physical) realization. The ontomathematical approach to gravitation is implicit in general relativity equating it to operators in pseudo-Riemannian space obeying the Einstein field equation and also well-known by the "geometrization of physics". Quantum mechanics shares the same by the separable complex Hilbert space and defining "physical quantity" by the Hermitian operators on it. One can interpret special Minkowski space involved by special relativity and the qubit Hilbert space of quantum information as Fourier counterparts immediately noticing that general relativity means gravitation as the Fourier counterpart of non-Hermitian operators implying non-unitarity and the violation of energy conservation and thus destroying Pauli's particle paradigm. Since the Standard model obeys it, this explains the impossibility of "quantum gravitation" in any framework conservatively generalizing the Standard model so that it would include gravitation along with electromagnetic, weak, and strong interactions. Einstein's geometrization of gravitation can be continued into a purely mathematical theory of it following Euclid's realization for geometry to be exhaustively built in a deductive and axiomatic way as well as Riemann's parametrization of all the class of Euclidean and non-Euclidean geometries by "space curvature", then being generalized to Minkowski space as the operators on pseudo-Riemannian space as the Einstein field equation means gravitation. The transition from mathematical gravitation to logical one can rely on the historical lesson of the pair of Lobachevski's and Riemann's approaches now "reversely", i.e., from the latter to the former. Logical gravitation is linkable to Hegel's dialectical logic and ontological dialectics abandoning their interpretations as a new zero logic substituting classical propionyl logic. The approach of ontomathematics generalizing that of ontology, traceable even to Aristotle's reformation of Plato's doctrine, needs Hegel's doctrine to be formalized as a first-order logic naturally containing Boolean algebra, isomorphic to both classical propositional logic and set theory being the class of all first-order logics, as a sub-logic along with Peano arithmetic as another sub-logic. The first-order logic at issue is called Hilbert arithmetic and elaborated in detail in other papers. It allows for both self-foundation of mathematics to be internally proved as complete and furthermore, quantum mechanics reinterpreted as quantum information to be included by the qubit Hilbert space interpretable in turn as a dual and physical counterpart of Hilbert arithmetic in a narrow sense, that is, both counterparts constitute Hilbert arithmetic in a wide sense, being mathematical and physical simultaneously and thus overcoming the Cartesian dualism of "body" gapped from "mind" by an abyss. Then, the proper philosophical interpretation of gravitation to be the fundamental ontomathematical force or interaction overcomes the ridiculous belief of the Big Bang wrongly alleged to

be a scientific theory. Ontomathematical gravitation suggests an omnipresent and omnitemporal medium of "God's" creation "ex nihilo" following only the natural necessity of quantum-information conservation particularly and locally manifested as energy conservation.

Keywords: classical quantum mechanics, dark matter, dark energy, dialectical logic, Einstein, energy conservation, entanglement, entanglement theory of gravitation, gravitation, Hegel, Hilbert arithmetic, Lobachevsky geometry, metaphysics, Newton, ontology and ontomathematics, Pauli's particle paradigm, quantum information, Riemann's space curvature, special and general relativity, the Standard model, unitarity

I. INSTEAD OF INTRODUCTION: LOBACHEVSKY'S DISCOVERY, BOTH PHYSICAL AND PHILOSOPHICAL REINTERPRETATIONS

The discovery of non-Euclidean geometry (whether by Friedrich Gauss, or by János Bolyai or by Nikolay Lobachevsky) was the first precedent of creating an alternative axiomatic system differing from the initial one by a single postulate, the famous Fifth Postulate of Euclid. Many alternative axiomatic systems have been constructed further, especially in the 20th century or in the area of mathematical logic: at that, not less consistent than their counterparts, historically preceding them. The resistance against the pluralism of new and new mathematical structures, described exhaustively, without any logical contradiction, was due to the prejudice that there exist privileged mathematical "reference frames", which are ostensibly fundamental for human knowledge.

Einstein's theory of relativity and the analogical relativism and pluralism of axiomatic systems in mathematics originated from the same (or at least similar) spiritual and intellectual milieu in that epoch featured by anti-dogmatism and the release from the chain of traditions. Human mind was emancipated. Mathematics obeys only human conventions which can be freely chosen as far they are consistent to each other. Indeed, some of those conventions seemed to be true by itself (e.g. such as Aristotelian logic, Euclidean geometry, or Newtonian physics), but this turned out to be only an illusion due to the tradition, a "QWERTY effect" without any other justification than conservatism and dogmatism (and maybe crucially, human conformism), because of the laziness of human thought, or Heidegger's famous "We [that is humankind] do not think yet".

Approximately, that is the usual philosophical and methodological interpretation about the importance of the discovery of non-Euclidean geometry, furthermore repeated many times, again and again, after which the connection with Einstein's special or general relativity is only superficial, an analogy or a metaphor, at the best originating from the shared intellectual "milieu" of the end of the 19th and the beginning of 20th centuries, and the emancipation from any spiritual or scientific authorities such as Aristotle, Euclid, Newton, etc., or from the traditions originating from them. The present paper does not reject that philosophical meaning of the discovery of non-Euclidean geometry: on the contrary, it confirms it once again, however radicalizing it in a way or degree far not trivial:

One can further trace the underlying structure of how gravity (such as Einstein's general relativity which describes it quantitatively and thus, as an abstract mathematical structure: respectively, as a property of pseudo-Riemannian space) can appear after the discovery of non-Euclidean geometry: at that, meaning the proper approach of Lobachevski, who tried to infer the Fifth Postulate as a theorem of Euclidean geometry by *reductio ad absurdum*. This means that Lobachevsky granted for the negation of the Fifth Postulate to be true as a premise, and thus as an axiom consistent with all the rest axioms of Euclidean geometry excluding the Fifth Postulate itself and hoping to deduce an obviously false corollary, by virtue of which and by virtue of *modus tollens* would follow that initial admission that the Fifth Postulate is true is false since that proposal contradicts the rest axioms.

However, Lobachevsky did not manage to find any contradiction, which his initial intention was. On the contrary, he realized a new geometry containing partly different theorems contradicting the corresponding theorems in Euclidean geometry; for example, the sum of the angles of a triangle is not "2π" (or it is greater than "2π" in Lobachevsky's proper new geometry since his admission was equivalent to the statement that there exist more than one, rather than none, line parallel to another line in a plain). Nonetheless, any contradiction did not appear though some theorems were different or even contradict to common sense's prejudice, but not formally and logically.

Then, one can consider an abstract structure (or rather, meta-structure) described thoroughly axiomatically consisting of all axioms of both Euclidean and non-Euclidean geometry, which, in fact, share all the rest axioms (which are 19 according to the axiomatics suggested by Hilbert in 1899), but the Fifth Postulate itself is "bifurcated" by its logical negation into two dual or complementary versions, which are further repeated by the interrelation of Euclidean and non-Euclidean geometries[1].

One may apply Husserl's "epoché", but now not to reality (as the original one was and is), and to the Fifth Postulate therefore occupying a skeptic and uncertain position to its statement about how many lines parallel to a given line exist in a plain. Thus, all the rest axioms of Euclidean geometry (and not less, those of non-Euclidean geometry for being the same), excluding the Fifth Postulate itself, to the pair of the Fifth Postulate and its logical negation or respectively, to the complete axiomatics of Euclidean and non-Euclidean geometries, share the same formal structure of a bit of information. The corpus of all shared axioms of Euclidean and non-Euclidean geometries corresponds to the state *before* either choice (usually notated as "yes" versus "not" or as "0" versus

[1] Euclidean and non-Euclidean geometries in Lobachevsky's original version or in Riemann's edition, then Minkowski's space interpretation of special relativity as a relevant "concave" modification of Euclidean space are reversely rethought physically after Einstein's "geometrization of gravitation" by general relativity in many enough papers such as: Sorli, Kaufman, Fiscaletti 2018; Brill, Jacobson 2006; Pitt, Schieve 2004; Jurdjevic 2001; Rowe 2001; 2001a; Fiore, Madore 2000; 1998; Corry 1998; Stevenson, Noss 1998; Boi 1996; Tagirov 1996; Toth 1993; Farwell, Knee 1990; Vargas, Torr 1989; French 1986; Vlasov, Logunov, Mestvirishvili 1984; Zund 1983; Portnoy 1982; Schein 1979; Torretti 1978; Pyenson 1977; Daniels 1975; Nickerson 1975; 1975a; Stein 1968; Rongved 1966; Pierpont 1923-1924.

"1"), and the pair of Euclidean and non-Euclidean geometries means accordingly the state *after* the explicit choice of either of both geometries.

Obviously, the reference to the axiomatics of Euclidean geometry and the Fifth Postulate is essential only historically being the first precedent for creating such a pair of two dual, complementary axiomatics in the sense explained in detail above. In fact, *the sketched structure is relevant to any pair of axiomatics built analogically.*

One can add to the same context Hegel's idea about "dialectical logic" or "ontological dialectics of development", opposed to classical logic meant to be propositional logic, that is classical Aristotelian logic. They differ from each other also by a single axiom: the "excluded middle" equivalent to the "noncontradiction rule" in the framework of the latter. Anyway, the two formulations are not necessarily equivalent, and the particular case of their equivalence is rather a special property of classical propositional logic. Dialectical logic rejects properly only the "noncontradiction rule" replacing it with Hegel's logical triad (or respectively, the general ontological scheme of development).

One can reveal still one similarity between Hegel's dialectical logic or dialectics and Hilbert mathematics[2] tending to a contemporary kind of Pythagoreanism. Both identify epistemology and ontology, or at least admit to share a common part. Speaking of the "triad of thesis, antithesis, and synthesis", Hegel related it equally well to both human cognition and ontological development not needing to distinguish them. Thus, dialectical logic obeys (at least in Hegel's intention) the requirement to be the generalized *objective* logic unlike Aristotelian logic relevant only to human knowledge (even not cognition if it is reckoned to be a process developing in time and thus obeying dialectics after Hegel). Analogically, Hilbert mathematics does not distinguish mathematical model and reality to which it refers.

However, Hegel's dialectical logic or dialectics is only a qualitative doctrine falling in Popper's definition of metaphysics being irrefutable in principle, able to explain any thesis including its antithesis, and thus without any practical relevance. On the contrary, Hilbert mathematics suggests quantitative descriptions and testable predictions.

II RIEMANN'S APPROACH

One can describe the formal and mathematical structure of the present paper as a morphism (supposedly even isomorphism or homeomorphism) of Lobachevsky's approach (being properly logical) into Riemann's one (being metrical and thus properly mathematical) to the same subject (which is the manifold of all non-Euclidean geometries after the generalization of the Fifth Postulate in a way able to unify it and its logical negation in a consistent way).

Then, an analogical morphism can be observed in relation to Minkowski space being a four-dimensional and "concave" generalization of Euclidean space and within which the parallel lines of the former correspond to concentric balls (which can be considered as generalized "circles" as well), so that the Fifth Postulate can be relevantly related also to Minkowski space after

[2] For example, in: *Penchev 2023 May 3*.

reformulating those parallel lines to be concentric balls. Indeed, even parallel lines in Euclidean space can be considered as concentric circles with *infinite* radiuses.

Pseudo-Riemannian space utilized to be the fundamental mathematical formalism of Einstein's general relativity refers to the "flat" Minkowski space (since the Fifth Postulate after the reformulation of parallel lines is equally valid to it) just in the same way as non-Euclidean geometries to Euclidean geometry. The same observation can be interpreted as a generalization of the later relation also to the case of any finite radius where that finite radius belongs to the *fourth*, imaginary dimension of Minkowski space.

Furthermore, the physical interpretation of the arbitrary and variable (including even differentially, i.e., from a point to another, infinitesimally close point) curvature of pseudo-Riemannian (thus able to be differentially described by the pair of space-time and energy-momentum tensors in any point as in the Einstein field equation) is gravitation after Einstein's general relativity, a theory being very well confirmed experimentally. Those space-time and energy-momentum tensors are conjugate, including in the sense of Emmy Noether's *first* theorem (1918) about the inherent link of conservation and symmetry. Of course, one can consider another pair of conjugate quantities, for example, the Hilbert-Einstein action versus a physically dimensionless quantity, which can be identified as quantum information (Penchev 2020 October 5).

Then, gravity can be reversely interpreted in an abstract and mathematical way: that is, as generating the *new dimension of finiteness to infinity*. The later seems to be inherently "flat" to the former, featured by the fourth, imaginary or "pseudo" dimension of both Minkowski and pseudo-Riemannian spaces. The physical sense of finiteness is locality[3], to which in particular any observation, experiment or experience refers or is to be related. Thus, gravity (by the mediation of its purely mathematical interpretation) can be identified to be the most fundamental physical force or interaction able to generate and really, omnipresently and omnitemporally generating the physical world and the universe following only mathematical laws and necessities, therefore removing the mythical "Big Bang" as a redundant and really semi-religious hypothesis about the creation of the universe "*ex nihilo*" as the state before the Big Bang might be qualified.

So, the creation cannot but take place by virtue of an only mathematical necessity, figuratively speaking, like "2+2=4". However, the historical lesson consisting in the relation of the approaches of Lobachevsky and Riemann to the construction of non-Euclidean geometries allows for the research of the origin to be returned still a step back, to logic, rather than from physics only to mathematics. The crucial assistance for the possibility of that next step back, in the origin of the universe and even that of the being at all is suggested by Riemann's idea for the parameter of space curvature to be introduced to the manifold of all non-Euclidean geometries, after which even

[3] There exist papers (e.g., Jung 2017; Wiseman 2006; Treacy 2003; Howard 1975; Selleri, Tarozzi 1986; Garuccio, Selleri 1980; Schiavulli, Selleri 1979; Selleri 1978) considering Einstein's concept of locality, but only as a physical one rather than interpreting it ontomathematically (as in the present paper). Rather Einstein's geometrization of electromagnetic field (e.g., Giovanelli 2016) then developed into the proper geometric theory of gravitation in general relativity is closer ant thus more relevant.

Euclidean geometry can be considered as a special particular case of non-Euclidean geometry with zero curvature, i.e., featured to be the single "flat" one.

Then, the morphism at issue (i.e., that from Lobachevsky's to Riemann's approach meant above) can be reduced to its underlying structure representing the transformation of a bit of information into an equivalent qubit of quantum information, consequently establishing that information whether quantum or classical is the same (or equivalent). The way of the interpretation of the pair of both Euclidean and non-Euclidean geometries as a single bit of information is already described. Now one is to demonstrate that the parameter of space curvature introduced by Riemann to unify Euclidean and non-Euclidean geometries is equivalent to a qubit:

What is sufficient is to be shown that there exists at least one bijection between the set of all possible values of a qubit (which are all pairs of complex numbers so that the sum of their modules is equal to a unit; that is, the values of a qubit can be described so: $\forall \alpha, \beta \epsilon C: \sqrt{|\alpha|^2 + |\beta|^2} = 1$), on the one hand, and the set of all real numbers (i.e., all possible values of Riemann's space curvature), on the other hand. That bijection is very well known a long time ago as the pair of Fourier transform and reverse Fourier transform. Indeed, the qubit variable can be reduced to a single one, the modulo of which is not greater than a unit (i.e., to any pair of real number in the close interval between zero and unit), and the later, real variable can be normed to the "variable of infinity", where that variable of infinity can be identified as an infinitesimal or differential variable usually notated as "dx" in relation to any variable "y" so that "$f(y,x) = \frac{dy}{dx}$" is the first derivative of the function "$y(x)$" to its variable "x".

There exist both Hamiltonian and Lagrangian representations in classical (also in quantum) mechanics. Then, one can borrow terms of "Hamiltonian and Lagrangian languages" (which can be further abbreviated to only "Hamiltonian" or "Lagrangian" just as one abbreviates Bulgarian language or English language to only "Bulgarian" or "English" accordingly) and utilize them analogically in mathematics rather than only in physic, from where they originate. The exact mathematical meaning of those "languages" is the following. Both refer to a pair of variables, the one of which is infinitesimal to the other: that pair is described in "Hamiltonian" if they are granted to be two independent variables, and in "Lagrangian" if they are subordinated as the function of a variable and the variable at issue, so that the first derivative of that function to the variable makes sense.

The admission that the two descriptions in both languages are equivalent is rather philosophical and thus it has to be granted as an axiom properly mathematically. Its sense is that any two variables can be equivalently considered whether as independent or as dependent correspondingly. Obviously, the condition is both variables to be defined on the same set. If one accepts a universal set, which can be further interpreted physically as the universe, any variables to any subsets of it can be considered as variables defined on the same universal set. Then the investigated axiom would mean that the two descriptions (the one of which grants the universal set, but the other not) are equivalent to each other.

In fact, the eventual accepting of the universal set does not mean the set of all sets implying Russell's paradox in relation to all sets not belonging to themselves neither the former implies the

latter because of the following reason. The universal set has to consist of "all things". So, if one grants that the sets are not things, the universal set would not imply that the set of all sets exists. However, and following the above new axiom underlying the equivalence of the description of both Hamiltonian and Lagrangian languages one can suggest that the universal set does not exist without touching the equivalence of the descriptions, after which one can admit not less consistently that the sets are things, after which the set of all sets is not implied again because the universal set itself is not now suggested. So, the eventual postulate that the description of both Hamiltonian and Lagrangian languages are equivalent is able to avoid Russell's paradox by a kind of epoché about whether the sets are things or not since the two descriptions granting either alternative are equivalent.

Meaning those preliminary notices, one can return to the problem of the morphism of a bit into a qubit, on the one hand, investigating it as the mutual "translations of Hamiltonian into Lagrangian or vice versa", and on the other hand, following the first historical precedent about the relation of approaches of both Lobachevsky and Riemann to the pair of Euclidean and non-Euclidean geometries. From the viewpoint of those mutual "translations" and speaking rather figuratively, one can say that both Hamiltonian and Lagrangian "languages" describe the same "gap" by two complementary ways, correspondingly: (1) by the two "shores of the gap" available as the two alternatives of any bit in information, i.e., in "Hamiltonian"; (2) by the continuous transition through the continuous "bridge over the gap" such as all values of any qubit of quantum information, i.e., in "Lagrangian".

One can immediately notice that the viewpoint at issue is borrowed from the justification of quantum mechanics, which is natural since the theory of quantum information including the comparison of a bit with a qubit is elaborated on the basis of quantum mechanics. Indeed, the main problem of quantum mechanics, from which many (if not all) of its "extraordinary peculiarities" originate, consists in how one to describe uniformly the discrete transitions of any quantum entity forced by the fundamental Planck constant and the continuous readings of the macroscopic apparatus described by the smooth differential equations of classical mechanics. The, the interlink of a bit (for quantum discreteness) and a qubit (for classical smoothness) is able to embody the problem at issue.

However, one can stare at the Cartesian "abyss" between Descartes's "body" and "mind", being in turn one of the most fundamental problems of philosophy in Modernity, seeing that the problem shares the same formal structure as that already resolved by quantum mechanics by introducing the separable complex Hilbert space and thus it can be mathematically interpreted to be the same, that is, isomorphic. This implies, that solution invented by huge efforts of many great minds for quantum mechanics in the beginning of the 20^{th} century is to fit to that main philosophical problem as long the two problems are granted to be isomorphic mathematically, which in turn suggests a new kind of Pythagoreanism, called quantum neo-Pythagoreanism in other papers (e.g., Penchev 2021 August 14).

Even more, one may observe that all problems about the foundations of mathematics culminating particularly in Gödel's papers (1930; 1931) share the same underlying formal

structure of the interlink of a bit and a qubit resolved by quantum mechanics before that, but formulated as a fundamental (or even as the most fundamental) problem by Cartesianism (but not in Descartes's proper philosophy) or classical German philosophy, etc. (even in Marxism or in "Marxism – Leninism" as the state totalitarian philosophy of socialism).

Indeed, the Gödel dichotomy about the relation of (Peano) arithmetic to (ZFC) set theory, both as first order-logics to propositional logic (that is, either incompleteness or contradiction) shares the same formal structure and thus can be resolved in a similar way to that already utilized by quantum mechanics and demonstrated in other papers as "Hilbert arithmetic" (e.g., Penchev 2021 August 14) and also implying some form of Pythagoreanism since the completeness of mathematics needs the external, as if non-mathematical world to be included within it, and then it can be identified with quantum neo-Pythagoreanism[4].

The addition of the pair of Lobachevsky's approach to non-Euclidean geometry compared with that of Riemann is really possible for sharing the same underlying formal structure, but the much more important observation is that the new viewpoint allows for building a bridge from the above three, already unified areas (quantum mechanics and information, philosophy in Modernity, and the foundations of mathematics) to the physical "force" or interaction of gravity by the mediation of Einstein's general relativity via pseudo-Riemannian space since it recounts Minkowski space in a way generalizing the pair of Euclidean and non-Euclidean geometries furthermore meaning both approaches of Lobachevski and Riemann to them.

III. EINSTEIN'S APPROACH

Einstein's approach to gravity can be expressed briefly and philosophically by the slogan "Physics is geometry"[5], which means that the physical theory of gravity, what general relativity is, can be interpreted as a geometrical theory, that of a special geometrical space, what pseudo-Riemannian space is. One can trace back the origin of the geometrical approach to gravity in special relativity.

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[4] On the contrary, the papers of Holton (1968) or Hon (2004), or Rindler (2009) can elucidate the deep link of Gödel incompleteness, Einstein's locality and Mach's empiricism.

[5] That "slogan" is only generalized to "Physics is mathematics" in the present paper since geometry historically had been physics (before Euclid) and has been mathematics (after him until nowadays). Sufficiently many papers investigate the "geometrization of physics" in the context of Einstein works (Giovanelli 2016; Wanas, Youssef, El Hanafy, Osman 2016; Hübsch 2015; Marcus 2015; Vishwakarma 2013; Hacyan 2009; Ungar 2008; 2005; Mermin 2005; Coleman, Korte 1995; Nakamura 1993-1995; Vlasov, Logunov, Mestvirishvili 1984; Daniels 1975; Guth 1970; Pierpont 1923-1923; Campbell 1922; Wrinch, Jeffreys 1921) or at all (Boi 2019; Pastorello 2019; Tavernelli 2016; Wanas, Youssef, El Hanafy, Osman 2016; Clemente-Gallardo, Hübsch 2015; Karamatskou, Kleinert 2014; Vishwakarma 2013; Kan, Shiraishi 2009; Marmo 2008; Cariñena, Clemente-Gallardo, Marmo 2007; Brill 2006; Chen, Ungar 2002; Rowe 2001; Brody, Hughston 1999; Fiore, Madore 2009; 1998; Olkhov 2009; 2007; Shojai, Golshani 1998; Coleman, Korte 1995; Nakamura 1993-1995; Ghaboussi 1993; Vargas 1992; Vargas, Torr, Lecompte 1992; Korotchenko 1990; 1990a; Maull 1990; Kibble 1989; Grosholz 1988; Kalinowski 1988; Pullin, Bressan 1987; Sparling 1986; Vlasov, Logunov, Mestvirishvili 1984; Prugovečki 1982; Daniels 1975; Nickerson 1975; 1975a. etc.).

Einstein formulated it initially and originally as an only physical theory corresponding to the very well-established experimental fact that the speed of light in a vacuum cannot be exceeded and this it is a fundamental constant of nature. Herman Minkowski (1908) demonstrated in a few years later after the original paper of Einstein (1905) that the physical theory of special relativity can be interpreted as a geometrical theory of a special vector space, which is called Minkowski space nowadays (e.g., Pyenson 1977).

Elaborating his theory of gravity as a generalization of special relativity, Einstein accepted Minkowski's geometrical interpretation and developed it further, in fact identifying Minkowski space as a physical medium, a modified "ether" of classical mechanics. Though special relativity dethroned (at least rhetorically) the "ether" and the absolute reference frame linkable to it, special relativity only replaced it really with the new "ether of light" by virtue of the absolute constant (and its unchangeability) of the speed of light in a vacuum to any reference frame.

Meaning that, the light cone literally and all Minkowski space generally can be identified as a physical "medium", corresponding to the ether of classical mechanics, but modified according to the postulate of the speed of light in a vacuum so that its deformations described by tensors analogical to those describing mechanical deformations in any tridimensional body in Euclidean space are the forces or interactions of gravity in fact and following the main ideas of general relativity. The parallel can be continued further:

Classical mechanics describes the deformations of tridimensional bodies in Euclidean space as processes in the course of time and thus equivalently as transformations in the four-dimensional Euclidean space, in which time is the fourth dimension obeying the additional condition of irreversibility, which can be justified only *ad hock* and *physically* rather than mathematically. Minkowski space can be interpreted as a simple "concave" analogue of the "convex" four-dimensional Euclidean space, in which only the dimension of time is changed to be "concave" rather than "convex" as the rest three, proper spatial dimensions of the four-dimensional Euclidean space.

Then, the following question is reasonable. Whether or how far can the "concaveness" of the dimension of time in Minkowski space represent the irreversibility of physical time? One may immediately notice that the dimension of time, though "concave" in Minkowski space, is anyway reversible as the rest three, "convex" and spatial dimensions. The only difference is that its reversibility is anti-isometric to that of the three spatial dimensions. Or in other words, if time only increases for its physical irreversibility, the spatial dimensions only decrease, which seems to be counterintuitive and contradicting the everyday experience where spatial dimensions are arbitrarily reversible in both directions for each of them.

Nonetheless, the spatial dimensions though arbitrarily changeable permanently decrease regardless of their change in both possible directions, but now in relation to the new absolute reference frame of the "light ether" introduced by special relativity to replace the immovable ether of classical mechanics. So, the "concaveness" of the dimension of time in Minkowski space represents well the irreversibility of time, on the one hand, and infers it from the postulate of not exceeding the velocity of light in a vacuum, on the other hand, though supplying also and

counterintuitively (but only at a first glance) the spatial dimensions with a similar or deducible irreversibility. One may say that the spatial dimensions are locally (i.e., in relation to any reference frame) reversible, but globally (i.e., in relation to the new "light ether" though there exists no reference frame linkable to it) reversible unlike the dimension of time being irreversible locally rather than globally so that its "concaveness" is able to represent its anti-isometry to the inherently "convex" spatial dimensions.

By the way, the anti-isometry of time to the spatial dimensions is consistent with quantum nonlocality or the phenomena of entanglement since the global reversibility of time can be interpreted as nonlocality at issue. On the contrary, Einstein's rejection of nonlocality as "spooky" actions at a distance would means that time is irreversible both locally and globally, and thus only the imaginary "half" of Minkowski space is able to make physical sense. Since quantum nonlocality and the phenomena of entanglement are very well established (including by the 2022 Nobel Prize for physics[6]) both imaginary and real subareas of Minkowski space make physical sense, and the dimension of time is thoroughly anti-isometric to the three spatial dimensions.

Then, one can return to Riemann's approach therefore reinterpreting Euclidean and non-Euclidean geometries as smooth deformations of the former variable from a point to point, and corresponding to different values of Riemann's space curvature from a point to another. Those continuous deformations of Euclidean space take place in the course of time, and if they are reinterpreted in a relativistic way, i.e., under the condition of the maximal speed of light in a vacuum, the four-dimensional Euclidean space, in which time is a "convex" dimension like the rest three spatial dimension, is to be replaced with Minkowski space and its "concave" time reflecting the postulate of the speed in a vacuum.

In other words, those deformations of Euclidean space, running in the course of the relativistic Einsteinian time rather than in the classical Newtonian time, are gravitation as it is described in general relativity. Indeed, one can trace back those relativistic transformations in the Einstein field equation where gravitation is understood as a tension in any point between the space-time tensor describing the "medium" of the relativistic spacetime, on the one hand, and the "force" of the energy-momentum tensor acting on that medium, on the other hand.

After that, the last and crucial deductive step is the reverse transition from Riemann's approach interpreted physically after Einstein to Lobachevsky's approach to the pair of Euclidean and non-Euclidean geometry and its logical meaning allowing for a physical reinterpretation of logic, in fact only continuing Einstein's approach to physics to be a special kind of geometry (namely that of pseudo-Riemannian space) in turn interpretable as a differentially changing, smooth, and relativistic deformation of Euclidean space, that is, replacing the slogan "Physics is a geometry" with another and still more generalized idea that physics is a logic via the intermediate representation for physics to be a geometry after Einstein's general relativity. One immediately notices that the development of the understanding of physics as a logic corresponds to a further "destruction" or "deconstruction" of the origin of general relativity from Riemann's approach to non-Euclidean geometry back in time, to that of Lobachevsky.

[6] Discussed in detail in: *Penchev 2023 March 13*.

Rather surprisingly and unexpectedly, one can see that the "physical logic" at issue is relevant to Hegel's dialectical logic only avoiding or overcoming its opposition to classical, Aristotelian propositional logic as a continuous "deformation" of the later in the course of time and furthermore therefore identifying that deformation of the "medium of propositional logic" as the physical force of gravitation after the mediation of pseudo-Riemannian space and accordingly, Einstein's general relativity. That is the crucial step of the present research, after which one is able to understand gravity as originating from Hegel's dialectics of contradiction and generating ontology as physics in the final analysis.

One can compare the new understanding of physics as a smoothly deformed "medium of propositional logic", which is really meant by Hegel's dialectics thus transforming logic into the ontology of the real physical world, with the foundation of mathematics, distinguishing propositional logic as the fundamental zero-order logic from set theory as the class of all possible first-order logics featured by different additional axioms, all of which vanish necessarily as to their class as a whole and thus allowing for the identification of propositional logic and set theory as the same structure of Boolean algebra, also expressible by the identification of any proposition with the set of all elements satisfying the characteristic property of that proposition.

Then, one can see any first-order logic determined by a certain tuple of axioms additional to the necessary tuple of the axioms of propositional logic as a deformation (thus representable as gravitation) of the initial, "flat" medium of propositional logic; that is, describing quantitatively Hegel's rather speculative "dialectic logic" only opposed to propositional logic as in Lobachevsky's approach now developing it further, following Riemann's pattern.

IV. FROM EUCLID'S GEOMETRY TO QUANTUM INFORMATION: VIA LOBACHEVSKY, RIEMANN, EINSTEIN. AND QUANTUM MECHANICS

Another and absolutely different viewpoint (to that for gravity to be understood as the deformation of the "medium of propositional logic" after and into Hegel's dialectic logic now newly understood geometrically as above) is that suggested by the only informational interpretation of quantum mechanics briefly called quantum information relied on the concept of quantum information measured in the newly introduced units of qubits therefore generalizing that of classical information measured in bits.

The essence of the newly approach of quantum information can be seen as originating from the main problem of quantum mechanics, namely: to unify the physical description of any quantum entity being *discrete* "by itself" for the fundamental Planck constant, on the one hand, and that of the same entity, but now measured by the readings of the macroscopic apparatus thus (in virtue of being macroscopic) obeying the *smooth* (and for that, *continuous*) differential equations of classical mechanics and physics. As this is very well known, that unification was historically accomplished by introducing the contemporary quantum mechanics justified by the separable complex Hilbert space and therefore unifying the inherent discreteness of Heisenberg's matrix mechanics and the analogically inherent smoothness (particularly implying continuity) of Schrödinger's undulatory (also called "wave") mechanics.

Indeed, one can assure that matrix mechanics can be interpreted as representing all quantum entities "by themselves" and discretely (by matrices, the rows and columns of which are enumerated thus discretely differentiated as within all columns or all rows between any column to any row); analogically, wave mechanics[7] represents them identically, but smoothly and continuously and thus by the readings of the macroscopic apparatus necessarily being smooth and continuous for the differential equations of classical mechanics. For doing this, the Schrödinger equation introduces a new physical quantity, "wave function" able to feature unambiguously the state of any quantum entity, being furthermore physically dimensionless and identifiable as some element (also called "point") of the separable complex Hilbert space. In other words, any wave function is seen from the viewpoint of set theory, after which the "axes" of the separable complex Hilbert space are granted to be an actually infinite set, but to which a certain finite matrix of the state after Heisenberg's version and after arithmetic (and the axiom of induction rather than after the set theoretical axiom of infinity, for example in its ZFC version) corresponds unambiguously.

Next, the bijection of all wave functions after Schrödinger and all matrices after Heisenberg (sketched in the last paragraph and the last sentence) is inherently embedded even in the definition of the separable complex Hilbert space by the identification of its "vector version" (for the finite and "arithmetic" matrix mechanics of Heisenberg) and its "function version" (for the actually infinite "set-theoretical" wave mechanics of Schrödinger). Thus, the separable complex Hilbert space is able to resolve the main problem of quantum mechanics: how to unify both viewpoints to the same quantum entity, namely that of it and "by itself" being necessarily discrete for the fundamental Planck constant and that of its representation by the readings of the smooth (and thus continuous) apparatus described by the differential equations of quantum mechanics.

Meaning its continuous "half", quantum mechanics introduces the concept of quantum information accordingly measured in special quantum bits, or abbreviated as qubits, since its discrete "half" may be interpreted to correspond to classical information measured in bits: indeed, any quantum leap is able to be liken to the jump between the two alternatives of a bit. Then, a qubit means the continuous (or even smooth) transition between the same two alternatives of a bit.

A qubit, "Q" is literally defined in quantum mechanics as the normed superposition of two orthogonal subspaces of the separable complex Hilbert space, or symbolically:

$Q \stackrel{def}{=} \alpha|0\rangle + \beta|1\rangle$ where α, β are complex numbers such that: $|\alpha|^2 + |\beta|^2 = 1$.

Since any two successive "axes" of the separable complex Hilbert space (that is: $e^{in\omega}, e^{i(n+1)\omega}$) are two orthogonal subspaces of it, the separable complex Hilbert space of quantum mechanics can be equivalently rewritten as the qubit Hilbert space of quantum information[8] (under a few additional, but rather technical conditions). Furthermore, quantum information measured in qubits can be equivalently interpreted as the generalization of classical information measured in

[7] The correspondence of Schrödinger and Einstein about the former's undulatory mechanics (Hanle 1979) can elucidate that the latter initially preferred it rather than Heisenberg's matrix mechanics associable with Bohr's viewpoint.

[8] In more detail in*: Penchev 2016*; etc.

bits, which relates to actually infinite sets or series[9]. In other words, classical information can be interpreted to refer to finite sets and thus, to arithmetic: unlike quantum information meaning actually infinite sets and thus relatable to set theory.

Then, one can notice that the main problem of quantum mechanics for unifying the discrete and continuous descriptions of mechanical motions or physical changes can be further related to the foundations of mathematics by the aforementioned mediation of the pair of classical information (for arithmetic) and quantum information (for set theory). Indeed, the qubit Hilbert space and before that, the separable complex Hilbert space overcomes the Gödel dichotomy about the relation of arithmetic to set theory by supplying arithmetic with an anti-isometric dual "twin" corresponding to the dual counterpart of either the qubit Hilbert space or the separable complex Hilbert space. So, quantum duality, or respectively complementarity is quite relevant to the foundations of mathematics or to the way for it to be complete following the model of quantum mechanics[10].

The crucial step for linking quantum information to geometry is the homeomorphism of the qubit Hilbert space and Minkowski space[11]. Indeed, still the separable real Hilbert space can be considered to be an arbitrarily-dimensional or respectively-dimensional generalization of Euclidean space (or vice versa: Euclidean space can be considered as the particular, tridimensional case of the separable complex Hilbert space), but this not yet sufficient for the present purpose. One needs just its complex analogue, better in its qubit version, for connecting with the four-dimensional "concave" Minkowski space, on the one hand, and to arithmetic, on the other hand.

As this is explained above, Minkowski space is sufficient to represent the postulate of not exceeding the speed of light in a vacuum featuring special relativity; or vice versa. In other words, the physical theory of special relativity and Minkowski space are equivalent once the fundamental constant of the speed of light in a vacuum is granted to determine the "light cone" of Minkowski space. That constant of light (regardless of its exactly determined value in our universe) can be also interpreted purely mathematically after the boundary of the areas of locality, incl. any possible unambiguously repeatable empirical experience or experiments, on the one hand, and nonlocality, ultimately established to be a legitime domain of physics (featured to be inherently probabilistical, and by Einstein's "spooky action at a distance") only after the 2022 Nobel Prize in physics for entanglement and quantum information[12], on the other hand, as that of infinitesimality to finiteness or respectively those of infinity to finiteness and of set theory to arithmetic, on the other hand.

Then, one can continue the parallel at issue meaning that pseudo-Riemannian space sufficient to describe gravity as a "geometrical force or interaction" and thus only mathematically after general relativity is only the arbitrarily and differentially from a point to another deformed "medium of Minkowski space" and thus described by tensors relevantly generalized, but

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[9] In more detail in: *Penchev 2020 July 15*.

[10] For that objective, one can introduce the concept of Hilbert arithmetic, in much more detail in other papers: *Penchev 2021 August 14*; etc.

[11] In detail in other papers: *Penchev 2022 February 4*.

[12] In detail in: *Penchev 2023 March 13*.

fundamentally analogical to those suitable for the deformation of any tridimensional body in Euclidean space and studied by deformation geometry for material science.

In fact, Minkowski space is no other than the propagation of a usual spherical light wave in a vacuum, i.e., in Euclidean space and interpretable as above to be the boundary of physical locality to nonlocality or that of infinitesimality to finiteness (respectively infinity to finiteness). Then pseudo-Riemannian space more than a century ago and very successfully utilized for the description of gravity is able and not worse to express a newly introduced field of entanglement, on the one hand and physically, but on the other hand and simultaneously mathematically, the current boundary of infinitesimality and finiteness (or: infinity and finiteness) as to any point of Euclidean space:

The latter representation means a kind of "mathematical field" (but quite not in in the exact meaning of the algebraic structure of "field"[13]) such that in any point the global Euclidean space exists a local Hilbert mathematics featured by a certain value of the parameter of the distance between infinity and finiteness[14] so that a mathematical force or interaction of gravity can be relevantly introduced as equivalent to the corresponding physical force or interaction and by virtue of the isomorphism of pseudo-Riemannian space whether interpreted physically or mathematically.

The aforementioned homeomorphism of Minkowski space and qubit (or separable complex) Hilbert space can assist for the assignment of "entanglement field" to the wider viewpoint sketched in the last paragraph. Indeed, and informally, the latter is the Fourier transformation of the former; as well as vice versa: the reverse Fourier transformation of the latter results in the former:

In other words, both spaces can be equally well represented to the same shared essence, but from the two opposite viewpoints embedded in both straight and reverse Fourier transforms: (1) the continuous and smooth changes current during the course of time and meat by classical physics or special relativity in Minkowski space; (2) the jump-like changes forced by the Planck constant and studied by quantum physics, in which the variable of time can be considered to be formally substituted by that of frequency as Fourier transform does or at least can be interpreted rather physically than mathematically.

So, if classical physics means only the viewpoint sketched in (1), quantum physics considers that conservative generalization unifying both (1) and (2) rather than considering only the newly introduced and complementary viewpoint forced by (2) though the latter is the prevalent accent in common sense's popular representation of quantum mechanics. Meaning that observation one can naturally question how the straight and reverse Fourier transforms should refer to the modified picture introduced by general relativity to describe gravity by pseudo-Riemannian space: that is, by arbitrary, but smooth deformations of the "medium" of Minkowski space in each point of it:

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[13] In fact one can trace the essential link of the standard understanding of field in algebra as a structure supplied with both additive and multiplicative commutative and associative operations featured furthermore by a single distribute law of the latter to the former, on the one hand, and the here introduced "mathematical field" by the rather complicated mediation of Hilbert arithmetic in both narrow and wide sense, but this would be far out of the subject of the present paper (however, maybe that of another in the future).

[14] Introduced and discussed in much more detail in other papers: *Penchev 2022 October 21*.

One can observe that pseudo-Riemannian space serves to split the representation of any smooth changes current during time in any spacetime point in two ones, covariant and contravariant, coinciding only in the particular case of zero gravitation. In other words, the detail account of the quantity of their mismatch is identified to be gravitation according to general relativity. One can accompany that the covariant and contravariant vector picture in any spacetime point are simultaneously available rather than complementary if one rewrites the pair of covariant and contravariant vectors by that of conjugate quantities in classical mechanics; that is, one needs entanglement or respectively, a spacetime field of entanglement to correspond to the split, but simultaneous representation by both covariant and contravariant vectors, and tensors as operators as to pseudo-Riemannian space.

Consequently, the conjecture advocated in the present and other papers (e.g., Penchev 2023 March 13) states that the fields of gravitation and entanglement are the same and corresponds to each other means the straight and reverse Fourier transforms. In fact, the proof of the statement seems to be rather elementary, even trivial and obvious: the identification of the pair of covariant and contravariant vectors by pseudo-Riemannian space, on the one hand, and the pair of any two conjugate vector variables by the qubit Hilbert space, on the other hand, is sufficient.

Then, one is to follow the pattern of the Einstein field equation, in which the interaction of gravitation is deduced to be a function of two tensor variables in pseudo-Riemannian space: the one is that of spacetime, and the other is that of energy-momentum in turn originating from the distribution and motion of all energies and masses in spacetime (and which are postulated to be the only source of gravitation by "Mach's principle" as it was called by Einstein[15]).

Next, one needs two entangled qubit Hilbert spaces (naturally corresponding to any two entangled, but arbitrarily remote quantum entities, such as photons, electrons, etc.) so that the entangled space-time of both corresponds to the curved spacetime in some finite neighborhood about any point of it, and the entangled energy-momentum of both, to the acting force or interaction into the same neighborhood about the same point just according to the Einstein field equation. One can immediately observe that Einstein's general relativity represents gravity as a local force or interaction therefore following the universal rule of classical physics also known by his picturesque and pejorative metaphor the "spooky action at a distance".

On the contrary, entanglement field means for gravity to be inherently nonlocal though represented equivalently by Einstein's local gravity. Then, one may integrate all over the nonlocal part of the universe (or respectively back in time) in order to obtain the Einstein gravitation acting

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[15] Einstein's "Mach principle" is discussed rather widely: for example, Dicke 2011; Newburgh 2007; Ne'eman 2006; Vigoureux, Vigoureux, Vigoureux 2003; Prasanna 1997; Nielsen 1987; Huang 1985; Okamura, Ohta, Kimura, Hiida 1975; Raine 1975; Higbie 1972; Katz 1967; Károlyházy 1964; Gürsey 1963; Brans 1962; Davidson 1957. Though Mach rejected both general relativity as well as Einstein himself to have followed his doctrine, the present paper is able to explain Einstein's "localism" (being inspired special and general relativity) as a generalization of Mach's worldview and philosophical ideas about physics. Their relations, both personal and theoretical, are also subject of many papers: de Waal, ten Hagen 2020; Rindler 2009; Hon 2004; Boi 1996; Montminy 1995; von Borzeszkowski, Treder 1993; Feyerabend 1984; Holton 1986; Zahar 1977, etc.

locally on the neighborhood at issue. That integration corresponds to common sense's opposition of the macroscopic Einstein gravitation versus the hypothetical quantum gravitation as belonging to the two extremes of the energetic scale: astronomical megascopic objects such as stars and nebulae, on the one hand, and microscopic quantum entities such as electrons and photons, on the other hand. This means that the gravitational interactions between stars and nebulae might result by a double integration: once, all over the nonlocal part of the universe, and then, all over the particles ("material points") constituting locally the star or nebulae at issue.

One should discuss the following ostensible contradiction between the nonlocal entanglement understanding of gravitation, sketched briefly above, and the usual local understanding of gravitation after Einstein. According to him, the source of gravity is both local and originating from the local masses and energies just corresponding to "Mach's principle", but according to Newton and in contemporary terms, it is both nonlocal and therefore violating "Mach's principle", even absolutely, after establishing only the option of nonlocal sources of gravitation and thus excluding any "light" masses and energies being inherently local. Finally, and rather paradoxically, gravity due to all or any nonlocal sources can be thoroughly equated to, and represented by the standard "light" energies and masses, the gravitation of which whether as bodies or as fields can be always demonstrated explicitly. The consideration in the present paragraph involves and thus needs "dark mass" and "dark energy" for its explanation.

The inherently nonlocal "dark mass" and "dark energy" (being implicit above) can be elucidated as the source of gravity and absolutely violating "Mach's principle": (1) it is the only source of "dark mass and darkenergy"; (2) the local change of its action onto different bodies, therefore being local in definition, is completely represented by the Einstein field equation; (3) the source of all or any nonlocal "dark mass and energy" is entanglement and thus quantum information or the change of probability (density or not) distributions from a quantum state to another all over the spacetime and that of the corresponding wave functions in the final analysis. So, the following conceptual equation able eventually to generalize the Einstein field equation (EFE) or "Mach's principle" for the crucial influence of "dark mass and energy" (DME) can be suggested:

(1) $EFE = \frac{d(DME)}{d(local\ spacetime)}$ where the local spacetime is all imaginary domain of Minkowski space, which special relativity means alone, therefore perfectly ignoring its real domain as it does not make any physical or empirical sense.

(2) $DME = emtanglement\ (nonlocal\ space\ time)$ where the local spacetime is all real domain of Minkowski space from where all physical actions at a distance originate though stigmatized to be "spooky" by Einstein and thus out of physics and even beyond science in the final analysis.

Granting the above "conceptual equation" and meaning the contemporary observations and experiment stating that DME is about 95-96% from the total mass and energy of the universe, thus, only 4-5% from the total mass and energy of the universe obeys EFE (respectively the former proposition from the "conceptual equation); as to "Mach's principle": (1) it is literally to refer only to those 4-5% at issue, featuring the "light universe" identified with all physics until now; (2) it is

to be generalized in order to comprise the rest “dark universe” by admitting quantum information as another legitime source of gravity and emphasizing that the suggestion implies the generalization of energy conservation (respectively, energy-momentum generalization) to a more general natural law of quantum information conservation therefore particularly involving the direct transformation of quantum information into DME for the explanation of the latter or the scandalous omnipresent and omnitemporal “creation ex nihilo” (in the sense of “creation from quantum information”) instead of the commonly accepted hypothesis of the Big Bang.

V. WHAT GRAVITY IS

So, the mediation of quantum information and the entanglement theory of gravity, on the one hand, and the link of quantum information to the foundations of mathematics by Hilbert mathematics in a wide sense, on the other hand, allows for the reinterpretation of gravity as a “mathematical force or interaction” featured by a certain value of the gravitational constant in our universe (also in a connection with the determined values of the Planck constant and that of the speed of light in a vacuum), but being not less relevant to any possible universe.

Furthermore, gravitation via mathematics and a kind of contemporary Pythagoreanism can be related to philosophy, and more especially to Hegel’s doctrine of dialectics, meaning which, gravitation can be realized as a “philosophical”, “dialectical” force or interaction generating the universe and the physical world at all, therefore suggesting particularly a proper philosophical explanation without the “Big Bang”, in turn being reinterpreted as the integral equivalent of the permanent, omnipresent and omnitemporal creation of the “light” and local universe from its prevailing counterpart of the “dark” and nonlocal universe, also interpretable as the transformation of the latter into the former or as a continuous process of decoherence really causing the universe.

The sketched idea for realizing gravitation can be detailed into successive stages, a few of which has been already accomplished historically in physics, mathematics, and philosophy and some do not yet, but the general framework suggested above allows for them to be predicted or at least admitted. The first of them can be additionally divided into two substages bifurcating and running in parallel in physics and mathematics, though its mathematical branch preceded historically and chronically its physical counterpart. Thus, the mathematical substage, being first, started from Euclid’s idea for an empirical science such as geometry to be rewritten axiomatically and deductively and thus as mathematical theory. As far as the ancient pre-Euclidean geometry was an empirical and possibly experimental science, it should be enumerated among physics if one follows the contemporary organization of cognition and thus opposed to mathematics in a modern sense.

Consequently, and from a contemporary viewpoint, Euclid’s innovation for rethinking geometry mathematically is revolutionary, but corresponding to the talweg of the original ancient Pythagoreanism though not literally, since Euclid’s mathematical and logical theory of geometry was not reduced to natural numbers, even sacral for the Pythagoreans, or to arithmetic in a more extended meaning. Rather, it can be linked to Aristotle’s logical and ontological reading of Plato’s doctrine, following the main idea of the former that logic allowed and allows for ontology therefore not distinguishing, but on the contrary, merging an empirical or experimental science such as the

ancient pre-Euclidean geometry with its revolutionary new, mathematical, deductive and axiomatic version suggested by Euclid himself onto the same science.

However, that unity and indistinguishability of physics and mathematics, accomplished by Euclid, contradicts the contemporary Cartesian organization of cognition, after which mathematics belongs to the mental domain of "mind" gapped from the material realm of "body" inhabited by physic. The gap at issue is an unsurmountable dualistic abyss rejecting any bridge able to overcome it. One can add that Descartes's dualism was intentional: in order to oppose the medieval scholastic Aristotelian doctrine of theology rejecting dualism or any duality as contradicting idea of the domination of God, but thus uniting and unifying the mental and physical worlds. Cartesianism[16] allowed for science and its empirical and experimental methods to survive and to emancipate themselves from religion and theology resulting in the modern look of the contemporary world and its exponential growth in the final analysis.

However, new scientific discoveries in the end of the 20th century and the beginning of the 21th century such as entanglement and quantum information, dark mass and dark energy, astronomical objects incompatible with the hypothesis of the Big Bang, as well as previous problems about the foundations of mathematics or quantum mechanics and information, philosophical theories such as Hegel's dialectic or Husserl's phenomenology, etc., have given impetus to research, among which the present investigation is to be situated. One might see them as restoring the medieval ontological worldview featuring Christian religion and ontology and even the ancient Greek Pythagoreanism, however in a way not to refute the scientific, social, technical and technological progress in Modernity ensued from the emancipation of science, technics and society from religion and theology. Even returning from the medieval ontological viewpoint, originating from Aristotle, to the preceding ancient Pythagoreanism is a more relevant qualification, which can be briefly concentrated in the neologism of "ontomathematics" instead of the commonly accepted of "ontology"

In other words, that "ontomathematical" worldview allows for the scientific explanation of entanglement, quantum information, dark matter and dark energy, the foundations of mathematics as well as many other problems, however at the cost of a philosophical revolution including the organization of cognition in Modernity, after which particularly mathematics does not build and suggest only models of reality, but studies reality by itself transforming itself into a philosophical discipline similar to way for logic to be considered as a part of philosophy after Aristotle's

[16] The later Cartesianism as well as notating the class of quite different modern Western philosophical doctrines presupposing fundamental dualism or endeavoring to overcome it are rather discernibly distinguishable from Descartes's original worldviews (e.g., Golumbia 2015; Shockey 2012; Rives 2009; Smith, Taylor, eds. 2005; Luft 2004; Esfeld 1999; Forbs 1997; Kasely 1996-1997; Funkenstein 1980; Larmore 1980; Schuster 1980) close to those of Newton and maybe originating from the 17th century's intellectual milieu. A series of papers researches his original doctrine, also inherently linked to his proper scientific works and both opposable and unifiable with Newton's (for example, Janiak 2013; 2012 Janiak, Sugden 2010; Smith, Taylor, eds. 2005; Crowell 2002; Slowik 1998; Gueroult 1980; Vigier 1993; Grosholz 1988; 1980 Gabbey 1980; Gaukroger 1980; Hacking 1980; Mahoney 1980; Iltis 1973; Cohen 1964).

revolution more than two millennia ago, but now by the mediation of contemporary mathematical logic interpreted to be simultaneously philosophical logic.

So, the present investigation considers a particular, but very important problem in the framework of the same fundamental philosophical shift. That problem consists in the realization of gravitation as a mathematical, even ontomathematical force or interaction able to create the physical world in virtue of a purely mathematical necessity therefore avoiding the quasi-religious and antiscientific conjecture of the Big Bang. That quite new and revolutionary understanding of gravitation, however, is not less a conservative generalization of Einstein's general relativity and thus consistent to it, but so that the newly introduced "ontomathematical gravity" includes quantum gravitation by means of an entanglement theory of it and the mediation of quantum information.

The premises of that synthesis about gravitation can be revealed by a kind of retro-analysis returning and restoring Euclid's revolution in the understanding of geometry from a physical and thus empirical science into a mathematical science following Aristotle's innovation at the same time regardless of having read the latter or not[17]. The hypothesis of whether an intellectual milieu in that ancient epoch in Greece or reversely, that Aristotle and Plato might have been influenced from geometry regardless of its version suggested by Euclid or before him are also consistent with the idea of the unification of physics and mathematics in geometry and individualized to be the "first substage" in the "first stage" after the present consideration intending to investigate gravitation as an ontomathematical force or interaction.

Meaning that unity of physics and mathematics in geometry before and after Euclid, one can reinterpret also non-Euclidean geometry after Lobachevsky compared with that after Riemann and his "space curvature". Obviously, the logical approach of Lobachevski to build a consistent geometry rejecting the Fifth Postulate followed rather Euclid's idea for geometry (or respectively, his own non-Euclidean geometry) to be a mathematical theory. On the contrary, Riemann's "space curvature" introduced a parameter, which can be understood to be physical, or both mathematical and physical simultaneously, so empirical observations or experiments are able to specify its exact value at any point of the real physical space.

That potential option in the essence of Riemann's innovative approach, was actually realized after Einstein's general relativity since the latter grants for the real physical space to be "curved", and even the curvature at issue to be identified with the source of gravitation or gravitation itself. As this is very well known, Riemann himself equated Lobachevsky's and his own approaches to non-Euclidean geometry. Just the proposal for equating them is especially important in the present context since it suggests also for geometry as a mathematical theory after Euclid to be identified with it, but as a physical theory before him and again a long time after hum and after Einstein's geometric (thus, in fact, mathematical) interpretation of the physical force or interaction of gravitation.

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[17] The interrelations of Aristotle and Euclid and eventual interinfluences between them are considered in a series of papers, for example: Humphreys 2017; Raymond 2014; Acerbi 2013; Pettigrew 2009; Høyrup 2002; Elden 2001; Greenberg 1988; Szabóo 1967; Apostle 1958; Greenwood 1952.

Then, one can reveal a new, heuristic and very instructive viewpoint to the history of Euclidean geometry and its Fifth Postulate culminated into non-Euclidean geometry after Lobachevsky or Riemann and furthermore into Einstein's geometric doctrine of gravitation. Geometry supports its unity of being a physical theory and a mathematical theory at the same time. Initially, it is a physical theory investigating properties and relations about plane figures and volumes. Then, Euclid revolutionized it rewriting it as a mathematical theory and demonstrating the physical and mathematical unity embodied in geometry. Euclid's enterprise was so successful that geometry was reckoned to be a mathematical discipline for about two millennia.

Following the same mathematical understanding of geometry, Lobachevski investigated as if the counterfactual option for rejecting the Fifth Postulate. However, he (and to his surprise) discovered an alternative geometric theory not less consistent than Euclidean geometry (rigorously proved by Riemann[18] and Poincaré[19] building a model of non-Euclidean geometry within Euclidean geometry). So, the finding of non-Euclidean geometry after Lobachevsky remained within the framework of the understanding of geometry as a mathematical theory imposed still by Euclid.

Riemann, anyway also interpreting non-Euclidean geometry as a mathematical theory, made the crucial step to restoring the unity, understanding it in addition as a physical theory, too. Indeed, the quantity of space curvature though introduced as a mathematical parameter in relation to the class of all non-Euclidean geometries relying on the rejection of the Fifth Postulate can be immediately interpreted physically; however, an option actually realized only after Einstein's geometric theory of gravitation granted to be an inherently force or interaction after Newton and his theory of "universal gravitation"[20].

However, Newton's work deduced the universal gravitation at issue was titled by himself "Philosophiæ Naturalis Principia Mathematica" (in English, "The Mathematical Principles of Natural Philosophy") so that the present investigation of gravitation as an "ontomathematical" force or interaction can be not worse interpreted as a continuation or even "resurrection" of Newton's original intention embedded in his own title. So, one can closely link the first and second substages of the first stage (according to the division above) if one accepts the viewpoint of the present paper about geometry historically unified physics and mathematics, tending to extend the unification started from physics further to logic and philosophy by the same mediation of geometry after Euclid's innovation to requalify it as a mathematical theory built axiomatically and deductively. Particularly, Newton himself understood his scientific undertaking to describe a physical force or interaction (such as the three fundamental ones meant by the Standard model

[18] Riemann 1854.
[19] Poincaré 1882; 1902
[20] Newton's conception of universal gravitation is widely discussed even nowadays: Slavov 2019; Cunha, Tort 2017; Nacer, Eddine 2016; Sim 2015; Lunteren 1993; Ducheyne 2011; 2009; 2006b; Nauenberg 2005; Tanona 2000; Onofrio 1998; Ihmig 1993; Dieks 1987; Cushing 1982; Waff 1976; Poultney 1971; Wilson 1970; Westfall 1967; etc.

nowadays), called by him universal gravitation[21], thoroughly and perfectly mathematically in the framework of philosophy, "Philosophiæ Naturalis" (in English, Natural Philosophy).

In fact, Newton's self-reflection on his work has been gradually removed by reducing it to a special investigation within the framework of physics alone, in which relevant mathematical models are only utilized, and the reference to philosophy is estimated to be wrong and archaic from the contemporary viewpoint to physics and the entire organization of cognition situated physics, mathematics, and philosophy quite differently, separating them absolutely from each other. However, that present day understanding of Newton's original work is anachronical. It relies on the philosophical worldview of Cartesianism, which had not been establish yet at that time. So, that contemporary interpretation is perfectly irrelevant to Newton's original intention and self-reflection. He did not elaborate a special physical theory as the greatest present-time physicists do: on the contrary, it created an ontomathematical work and doctrine, in fact restored or continued partly as an undertaking in the present paper.

That resurrection of Newton's own approach is a key for a relevant reinterpretation of Einstein's general relativity usually seen to be a conservative generalization of Newton's physical theory of universal gravitation. Indeed, our age by its scientific common sense means Einstein to be a modern physicist therefore elaborating special physical theories involving relevant mathematical models, containing verifiable predictions thereafter confirmed by all or almost all corresponding experiments. Then, any philosophical reflection on Einstein's theory of gravitation can be only indirect and immaterial to it since his general relativity cannot be an ontomathematical doctrine as any physical theory in Modernity cannot be that.

However, if one reflects on Einstein's theory as a continuation of Newton's undertaking after his own self-reflection for his tractate to be ontomathematical, then, the former may be also realized as another ontomathematical doctrine, utilizing geometry to express its ontomathematical ideas since geometry had been burdened to be a mediator between physics, mathematics, and philosophy still since Plato, Aristotle and Euclid's age. One can interpret the present study quite modestly and humbly: as an ontomathematical explicit elucidation of Einstein's approach being ontomathematical by itself, but implicitly: anyway, only situated in a relevant historical and philosophical context (as here), its real ontomathematical intention is able to stand out and to be outlined.

That context has to include the proper philosophical dialectics and dialectical logic of Hegel, after which and after Riemann's innovation to non-Euclidean geometry can be inherently linked to Einstein's reading of Newton's gravity by pseudo-Riemannian space, a special, metric and vector geometric space generalizing Minkowski space in turn generalizing Euclidean space in a way relevant to its transformation into non-Euclidean space. Hegel's dialectical logic and especially involving it in relation to geometry including the geometrically interpreted physical

[21] The relation of the three fundamental interactions meant by the Standard model with Newton's universal gravitation generalized by Einstein's general relativity is discussed in certain papers; for example: Deur 2019; Arbuzov, Barbashov, Borowiec, Pervushin, Shuvalov, Zakharov 2009; Jones 2009; El Naschie 2005; Saller 1998.

gravitation after Einstein's general relativity is enumerated to be the second stage after the geometrical and physical first stage correspondingly divided into the two substages described in more detail above. Now, Lobachevski's logical approach to non-Euclidean geometry is crucial after Riemann's space curvature and Einstein's link from that space curvature to gravitation.

Indeed, one can generalize the approach of Lobachevsky as applicable to any consistent axiomatics and any axiom in it. So, the idea is that one can always consider the pair of axiomatics sharing the same tuple of axioms excluding a single one, but under the additional condition that the one means the statement (such as the Fifth Postulate in Euclidean geometry), but its counterpart (i.e., the other member of the pair) does its logical negation (such as the negation of the Fifth Postulate replacing it into non-Euclidean geometry). Then, a parameter (such as Riemann's space curvature in the case of the pair of Euclidean and non-Euclidean geometries) is always able to unite the pair of two axiomatics following the pattern established by Riemann himself to the particular case of both Euclidean and non-Euclidean geometry.

Furthermore, one can notice that the pair of two dual axiomatics as above models very well Hegel's "dialectical" contradiction in a way absolutely consistent with classical propositional logic since the two axiomatics are complementary or dual to each other in definition: their simultaneous consideration is forbidden just as the simultaneous measurement of any two conjugate quantities of the same quantum entity in quantum mechanics (meaning also the option for it to be entangled with another quantum entity and which will be discussed in more detail below).

The difference of two dual axiomatics as above, so that both obey classical propositional logic, in relation to the introduction of any paraconsistent logic for the description of Hegel's "dialectical contradiction" is worth to be emphasized. The pair of two dual axiomatics means two dual first-order logic, two mathematical theories about the same structure, but inapplicable simultaneously to it. On the contrary, any paraconsistent logic is meant to be a "zero-order" logic repairing classical propositional logic in a way to be able consistently to deal with propositions containing logical contradictions and therefore violating the "noncontradiction rule".

The model of two dual axiomatics as two first-order logics is borrowed from quantum mechanics forced to described consistently a certain natural and physical "dialectical contradiction", namely "wave-particle duality" or the uniform description of discreteness (for any quantum entity due to the fundamental Planck constant) and continuity (for the readings of the apparatus measuring the same quantum entity). Though there existed and exist ideas for "quantum logics" to replace propositional logic to all quantum entities as those eventual zero-order logics relevant to them, quantum mechanics introduces (in its "mainstream") a specific structure, the separable complex Hilbert space, thus a first-order logic relevant to all quantum entities or to quantum mechanics studying them.

Hegel himself by interpreting his doctrine of dialectics as a new and "ontological logic", "dialectical logic" directed to be searched for a relevant new zero-order logic able to deal with "dialectical contradictions". Hegel's original doctrine in its whole is not "revolutionary", so that his "dialectical logic" should be rather a conservative generalization of classical propositional

logic so that the “noncontradiction rule” to be able to be inferred as a particular case from the universal law of dialectical contradiction postulated by dialectics.

Hegel’s doctrine nowadays is rather more known by its Marxist “revolutionary” reinterpretation embedding it in the philosophy and practice of state socialism in the USSR and its satellites in the 20th century. Particularly and relevantly to the present context, that reinterpretation implies for the understanding of dialectical logic to be a revolutionary, i.e., non-conservative generalization of dialectical logic therefore excluding all previous knowledge including mathematics and physics as far as all of them use propositional logic or can be considered as first-order logics. In fact, Marxism or Marxism-Leninism as the state philosophy in the totalitarian socialistic countries did not support so extravagant and radically revolutionary thesis: they utilized almost all corpus of the previous knowledge conservatively and quite standardly. So, the Marxist suggestion for dialectical logic as a non-conservative generalization of classical propositional logic was really rejected by *reductio ad absurdum*, since the almost entire corpus of all preceding cognition should be refuted including all technologies and technics because they rely on the knowledge in turn relying on classical propositional logic: an omnipresent “cultural revolution” should take place, but the states which really started that “cultural revolution” such as the People's Republic of China in Mao’s age or the Khmer rouge’s regime in Kampuchea ended in disaster as well as the USSR itself, though its revolutionarity was much more restricted: only in social science and practice.

So, the moderate original understanding of dialectical logic as a conservative generalization of classical propositional logic is confirmed by *reductio ad absurdum* since the alternative Marxist “revolutionary” hypothesis for dialectical logic as a non-conservative generalization led to absurdity and social crashes. However, that moderate Hegelian understanding of dialectics is indistinguishable from its interpretation as a specific first-order logic, for example that utilized by quantum mechanics introduced the inherently dual separable complex Hilbert space. Indeed, the conservative generalization means that all axioms of propositional logic are kept only complemented by specific “dialectical” one or more axioms. Then, the axiom(s) at issue can always feature a relevant first-order logic just as quantum mechanics really does. As well as vice versa, the approach of quantum mechanics recognizing complementarity and duality in the framework only in its own subject can be immediately and consistently reinterpreted to be a relevant dialectic logic in Hegel’s conservative manner and thus transferable in any other area of human cognition, as long that area needs dialectical logic.

That rather radical interpretation of the specific first-order logic utilized by quantum mechanics only to its own subject to be a relevant “dialectical” logic for any other domain needing “dialectical logic” is not commonly accepted: it is rather a contribution or proposal featuring the present or other my papers (e.g., Penchev 2023 January 3). Nonetheless, if it is granted, the unity of Lobachevsky’s approach to the pair of Euclidean and non-Euclidean geometries with Riemann’s one can be almost trivially generalized to any case where the separable complex Hilbert space nay be involved as a relevant first-order logic therefore indistinguishable from the Hegelian

conservative interpretation of dialectical logic and further pioneering the shift to the ontomathematical understanding of gravitation intended to be advocated here:

In other words, the conjecture is that any dialectical contradiction exemplified by the pair of Euclidean and non-Euclidean geometries can be constantly supplied with a real variable (or parameter such as Riemann's space curvature) further implying the cherished "ontomathematical gravitation" after Einstein's general relativity. Speaking loosely, one may state that any dialectical contradiction implies a physical force or interaction indistinguishable from gravitation, as long it is interpreted to be ontomathematical, so that the Cartesian "body" and "mind" are unified and the origin of reality (instead of the anti-scientific semi-religious "Big Bang") from that conservative Hegelian dialectical logic (as if "ex nihilo") can be investigated in a rigorous and inherently scientific way turning out to be necessarily "ontomathematical".

Furthermore, the conjecture suggested in the last paragraph above can be modified or generalized so that the model of any "dialectical contradiction" can be built absolutely consistently in pseudo-Riemannian space being in turn a generalization of the pair of Euclidean and non-Euclidean geometries after the addition of the boundary of mathematical infinitesimality versus finiteness (being equivalent to the boundary of physical locality versus nonlocality) where the boundary at issue can be exemplified by the fundamental constant of the speed of light in a vacuum. As a direct corollary from that conjecture is that any dialectical contradiction would result into a physical force or interaction indistinguishable from gravity, therefore proving that gravitation is an ontomathematical force or interaction, which is the proper subject of the present paper.

Nonetheless, the conjecture at issue (here only granted) is proved in other papers[22] (though not in an explicit way which can be the intention of a future article) by involving the newly introduced both scientific concept and mathematical structure of Hilbert arithmetic in both narrow and wide senses and demonstrating that is mathematically complete therefore overcoming the Gödel counterargument in any modification originating and inferable from it. However, Hilbert arithmetic implies further Hilbert mathematics[23] opposed to Gödel mathematics being inherently ontomathematical, i.e., including all the physical world and thus physics in its framework. For its ontomathematical claim, any "dialectical contradiction" turns out to be within it, on the one hand, and its mathematical "model" (using the customary, but incorrect "Cartesian slang" of today's philosophy and science) is also and necessarily within it, on the other hand, so that gravitation can be introduced as that ontomathematical force or interaction by which the doubled ontomathematical reality is bifurcated into the "contravariant" Cartesian reality by itself and its unambiguous "covariant" counterpart, again utilizing the habitual "Cartesian slang": about "mathematical models". So, aforementioned hypothesis means physical gravity to be an ontomathematical force or interaction linking the Cartesian "model versus reality" (originally, the gapped "body" and "mind"), therefore organizing a bridge between them needing the unity of mathematics, physics, and philosophy for its existence.

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[22] For example, Penchev 2023 May 3.

[23] Also discussed in a series of papers such as: *Penchev 2023 May 3; 2023 March 13; 2023 January 3 2022 October 21*; etc.

Then, one can consider in detail the reflection of the bridge (above a metaphor, but meant in an absolutely rigorous and exact scientific significance by "Hilbert mathematics") back onto quantum mechanics, and more precisely, onto quantum information, which can be also interpreted as an entanglement theory of quantum gravitation. In other words, but speaking loosely, entanglement is gravitation represent at the quantum level, and not less, gravitation after Einstein's general relativity is entanglement measurable by the apparatuses obeying classical physics and thus being inherently macroscopic.

That idea also is described in much more detail in a series of papers[24] and now only its "ontomathematical projection" will be investigated. Indeed, if gravitation is proved to be an ontomathematical force or interaction, and entanglement is its equivalent at the microscopic quantum level, one dare state quite reasonably that entanglement causes quantum reality, that is physical reality observed at the quantum level and the studied by classical quantum mechanics as ostensibly "ready in advance", or speaking figuratively: "prêt-à-montre".

Then, one can trace (historically, back) how entanglement, being simultaneously gravitation at quantum level (and thus "quantum gravitation" in fact) is able to generate thoroughly all the rest three fundamental interactions meant by the Standard model and to underly it therefore explaining it. The idea is discussed in detail in other papers (e.g., Penchev 2023 March 13), so that it will be now sketched quite briefly. One can generalize the concept of "reference frame" being fundamental for general relativity in order to be introduced a still more general principle of relativity[25] called "quantum relativity" and requiring for all physical laws to be invariant to the newly introduced discrete (or "external", or "quantum) reference frames situated to be in any discrete relative motion (such as any quantum leap) to each other rather than in any smooth relative motion featured by a corresponding mutual acceleration as Einstein's principle of general relativity needs. If one has granted in advance both quantum relativity and the equivalence of entanglement (at a quantum level) and Einstein's gravitation (at both macroscopic and astronomic level), the equivalence of entanglement and the rest three fundamental interactions according to the Standard model (namely: electromagnetic, weak and strong) follows immediately. The main idea involving the bijection at issue consists in the following:

Any reference frame in a certain relative motion to another and meant by general relativity can be unambiguously determined in any point of its relative trajectory by its position, its velocity granted to be a variable dependent on the variable of position as its first time derivate, and also by its acceleration in turn granted to be a variable dependent on the variable of velocity as its first time derivative as well. Obviously, that description of any reference frame admissible in general

[24] Penchev 2023 March 13, etc.

[25] Further generalizations of the fundamental principle of relativity according to Einstein's theory of gravitation as well as different aspects of the latter relevant to the advocated here worldview are meant, for example, in the following papers: Pawlowski, Papoyan, Pervushin, Smirichinski 1998; Frampton, Nielsen 2019; Fox 2016; de Felice, Preti 2009; Tresoldi 2009; Mensky 2004; Shirafuji, Nashed, Kobayashi 1996; Reuse 1984; Goded 1975. The viewpoint to a still more general principle of relativity able to be relevant to quantum, i.e., discrete mutual motions of reference frames is developed in much more detail in another paper: *Penchev 2021 June 8*.

relativity uses Lagrangian (language) and the question of how the same would look like, being translated in Hamiltonian (language), is natural. The hypothesis advocated in detail in the cited papers suggests that the newly introduced discrete or external reference frames represent in fact the Hamiltonian "translation" of the concept of a relatively and arbitrarily accelerated reference frame described by general relativity only in Lagrangian (language). So, if both languages are absolutely equivalent to each other and the aforementioned premises are also valid, this implies the researched equivalence of entanglement and the rest three fundamental interactions according to the Standard model.

Then, the position will be exhaustively represented by the four spacetime coordinates independent of each other, and velocity, accordingly, by the pair of it and still one position and its corresponding four spacetime coordinates, absolutely independent of the former just as Hamiltonian (language) needs. As to acceleration in Hamiltonian (language), it in turn doubles the former by its counterpart featured also by an octave of space time coordinates just as the former. Totally, 16 spacetime coordinates are sufficient for any reference frame of general relativity to be translated unambiguously into Hamiltonian language. One can immediately see an analogue with the 16 most fundamental particle (i.e., all but without the Higgs boson), which can be justified by the following reason:

One can introduce the concept of the absolute reference frame of the universe, and that absolute reference frame can be visualized by the metaphor of the relative reference frame of the universe to the absolute reference frame of the "luminal ether" newly introduced by special relativity and then transferred into general relativity with the arrangement that the latter cannot be defined rigorously and consistently, because of which the absolute reference frame of the universe is to be postulated rather than inferred as a relative reference frame to the "really absolute" reference frame linked to the light ether (since that is not a scientific notion), and then only loosely explained by the figure of the "light reference frame".

There exist also two very important clarifications connected immediately to the newly introduced concept of the absolute reference frame of the universe. They are correspondingly: (1) the invariance of all physical laws to the visible (local) and invisible (nonlocal) universe; (2) the hypothesis of the "Big Bang" which is ostensibly the real beginning of the universe.

The former problem can be loosely represented as: what happens when (or where in spacetime) any invisible part of the universe becomes visible to us? Do the physical laws remain the same? Or is the transformation from invisibility into visibility able to change certain physical laws in general? Obviously, the intuitive answer of scientific common sense would be: "No, whether the invisibility or the visibility of any physical entity might not influence the physical laws relevant to it. Furthermore, that is a great philosophical problem that existed a long time before Kant, but articulate especially discernibly by himself: do the things 'by themselves" ("Dinge an sich") and "for us" are the same. His solution is revolutionary, "Copernican": the things are only "things for us" and thus the "metaphysical" problem about the "things by themselves" is meaningless, respectively one ought to postulate the identity of anything both "by itself" and "for us" therefore

particularly confirming the invariance of all physical laws of both visible and invisible parts of the universe.

The historical context of Kant's innovation was the emancipation of science (first of all, empirical and experimental science) from religion and theology too busy to resolve abstract and metaphysical problems (where one's authority, e.g., that of Aristotle, is crucial) neglecting and excluding any contradicting empirical and experimental proofs. Kant's "Copernican revolution" suggested an opposite solution following an abstract and philosophical approach also confessed by the theologians but implying conclusions antithetical to theirs. However, the context of the present and other papers (e.g., Penchev 2023 March 13) tends to identify to each other: (1) the physical opposition of locality (visibility), nonlocality (invisibility); (2) the mathematical opposition of infinity (infinitesimally) and finiteness; and (3) the Kantian opposition of the transcendental and the transcendent. So, the problem seeming to be only philosophical turns out to be simultaneously and indivisibly both physical and mathematical and thus depending on which mathematical problem is meant or on experiments and empirical observations. That can further be interpreted to be a generalization or radicalization of Kant's worldview: called "scientific transcendentalism" in other papers (e.g., Penchev 2020 October 20).

Nonetheless, the same problem arose in quantum mechanics where some extreme interpretations admit that the experimenter's measurement ostensibly determines the real measured quantities of the quantum entity, or by the corresponding speculative philosophical reflection, as if the subject's observation creates the observed object, by the by, contradicting Kant's original transcendentalism. However, the so-called "Copenhagen interpretation" is very close to a quantum reading of Kantian doctrine: the problem about quantum entities "by themselves" is meaningless. If one needs them, they should be postulated to be identical with their measured counterparts.

In fact, that postulation is implemented in the separable complex Hilbert space granted to be the basic mathematical structure of quantum mechanics after unifying Heisenberg's matrix mechanics and Schrödinger's undulatory mechanics. Indeed, the former can be related to the vector interpretation of Hilbert space (implying matrices for the description of any vector transformations), on the one hand, and to the discrete changes of quantum entities "by themselves", on the other hand. Respectively, the latter refers to the functional interpretation of Hilbert space (indeed, wave function is a function in the rigorous mathematical meaning of functions relevant to Hilbert space), and to the readings of the macroscopic apparatus obeying the smooth differential equations, on the other hand. So, the separable complex Hilbert space, in turn being a mathematical structure as well as the mathematical foundation of quantum mechanics, embodies Kant's transcendental solution furthermore specified by the Copenhagen interpretation, for example.

So, one can notice that the boundary of visibility, locality, and empirically or experimentally accessible experience, on the one hand, in relation to invisibility and nonlocality, on the other hand, is represented by two absolutely independent approaches, correspondingly in quantum mechanics and relativity, both special and general. The conjecture that they can be unified since both mean the same only expressed in two alternative ways is natural. Furthermore, that unification can be

interpreted to be still one viewpoint (as well as a method) to quantum relativity justified quite differently in other papers (e.g., Penchev 2023 March 13).

The idea suggested in the above paragraph can be illustrated by the physical theories of both quantum mechanics and relativity and their basic mathematical formalisms (i.e., the separable complex Hilbert space and Minkowski space, resp. pseudo-Riemannian space). The former means the identification of the propagation of light in a vacuum in the empirical and physical space with quantum measurement. That identification seems to be absolutely counterintuitive, even absurd to scientific common sense since the propagation of light in vacuum and relativity as its relevant theory is usually granted to be a macroscopic phenomenon and thus opposed to quantum measurement being inherently microscopic. However, the approach to quantum gravity by means of quantum relativity suggests quantum measurement to be discussed in terms of the exchange of light signals between the measured quantum entity (and an eventual hypothetical "observer" linked to it and its reference frame) and the macroscopic experimenter's reference frame (which may be visualized also by the reference frame of the apparatus supplied by an observer, namely the experimenter at issue).

Then, one can invent the following "thought experiment" relevant to quantum relativity following the talweg of Einstein's famous "Gedankenexperimenten", even more so the "observer within the measured quantum entity" can be imagined only mentally. Thus, the experimenter sends a light signal questioning about the state of the reference frame supplied by its microscopic observer able to answer by a response light signal, in which he or she has encoded the state of it reference frame to message it to the macroscopic experimenter following the reading of the apparatus. However, one is to mean Heisenberg's uncertainty since the size of the alleged macroscopic observer is commensurable with the Planck length, time, and mass derivative from the Planck constant together with the other two most fundamental constants.

Speaking loosely, though the experimenter asks classically, that is, for example, for both position and impetus, her or his microscopic "colleague" is able to answer only to either: either position or impetus (respectively, any superposition of them obeying Heisenberg's uncertainty). If the macroscopic experimenter himself or herself has specified the question complying with the requirements of Heisenberg's uncertainty by the relevant preparation of the experiment, the microscopic observer's answer would be absolutely exact by relating only to the precisely formulated question therefore admitting an unambiguous answer. What is the important is that the discussion in terms of the exchange of light signals once Heisenberg's uncertainty is meant in advance is quite relevant and similar to the descriptions containing in Einstein's original "Gedankenexperimenten" so that the bound between visibility and invisibility for the propagation of light in a vacuum, on the one hand, and for any quantum measurement can be equated but under the additional condition for the macroscopic experimenter's interrogation to be bifurcated strictly restricted by Heisenberg's uncertainty.

Now, the assistance by the unification of the mathematical structures relevant in each theory can be very useful as far as the macroscopic experimenter's questions in terms of relativity (formally, in terms of special relativity, but in a way able to be immediately generalized to general

relativity), but her or his microscopic "colleague" responses in terms of quantum mechanics needing only the addition of Heisenberg's uncertainty. So, the former is to use Minkowski space (eventually generalized to pseudo-Riemannian space if need be), but the latter answers in terms of the separable complex (respectively, qubit) Hilbert space featured by duality. Nonetheless, both can understand each other very well as long Heisenberg's uncertainty is meant by both. That uncertainty is inherent for the microscopic researcher, but what about the macroscopic colleague? For the dialog to be meaningful, he or she is forced to interpret the dual Hilbert space it terms of Minkowski space (or in those of pseudo-Riemannian space as long need be), after which the dual counterpart fits exactly to the real domain of Minkowski space alleged by Einstein to be (ostensibly) physically meaningless. Now forced by the communication with the quantum colleague, the macroscopic experimenter should reject Einstein's prejudice about the "spooky action at a distance" though the refutation to Einstein's viewpoint is not necessary, in fact, if general relativity is also involved as this will be demonstrated a little below.

First, for certainty, the macroscopic experimenter rejects Einstein therefore identifying the dual Hilbert space with the real domain of Minkowski space and allowing for the latter to be physically meaningful. In other words, the researcher suggests that the half interrogation to the microscopic colleague can be interpreted physically to be nonlocal and the response cannot be received by the exchange of light signals. For example, and following literally Einstein, the two observers at issue establish their temporal simultaneity. Then, the answer about the energy of the quantum entity supplied by an "observer" would be nonlocal and impossible for any resolution by light signals.

Secondly, the macroscopic experimenter can anyway involve Einstein's gravitation by means of pseudo-Riemannian space in order to demand for the nonlocal counterpart of the response to be representable locally. Indeed, pseudo-Riemannian space in each point can be decomposed into the two domains of Minkowski space where the pair of the "real" and "imaginary" (areas of Minkowski space) corresponds to the pair of "covariant" and "contravariant" (for pseudo-Riemannian space). Then, the nonlocal half of the quantum colleague's response can be anyway represented locally for the macro-experimenter since both real and imaginary domains of Minkowski space, partly "overlapping" each other, are locally accessible to him or her. So, the energetic half of the response can be observed locally as a corresponding quantity of gravitational energy due to the curvature of pseudo-Riemannian space in turn because of the overlapping of the real and imaginary domains of Minkowski space. The quantum investigator can translate the macro-colleague's gravitational observations in the language of the separable complex (qubit) Hilbert space as phenomena of entanglement where two dual Hilbert spaces partly overlap each other according to the degree of entanglement. In other words, the micro-researcher will conclude that gravity (for the other colleague) is entanglement at the own quantum level as well as vice versa. This means: quantum gravity is entanglement.

The quantum colleague, "Alice", can also undertake an attempt for communicating with the macroscopic physicist, "Bob", sending qubits, a single qubit is enough, rather than light signals as above where the initiative belongs to the latter. This justifies the name "Alice" and "Bob" since it exemplifies, in fact, quantum communication, where those "names" are commonly accepted. So,

Bob receives Alice's qubit instantly however without two oppositions (incorrectly granted to be two bits though they constitute a single bit as this is elucidated in detail in other papers: *Penchev 2021 July 8*; etc.). Both can be transmitted only by a classical channel obeying the postulate of light locality. This states the so-called "teleportation theorem". Bob can immediately interpret the missing bit of classical information as follows. It should refer to which domain of Minkowski space is meant, or respectively, to which member of the pair of conjugate quantities (such as "energy" and "time") the transmitted quantum information relates. The two oppositions of the missing bit can be expressed so: (1) "Alice" versus "Bob"; (2) if the case is "Bob", which domain of Minkowski space: "real" versus "imaginary". This is a literal reading of the "teleportation theorem" after the case at issue.

Now, the conjecture about quantum gravity as entanglement would be added. Then, Alice sends her qubit identically as in the former case only interpreting it differently: that is, as meaning the entangled state of both conjugate quantities simultaneously. The mathematical sense is the following. Any qubit can be equivalently interpreted as an actually infinite series of classical bits (in detail, in: *Penchev 2020 July 15*), to which a single bit can be complemented absolutely "painlessly", i.e., without changing the quantum information contained in Alice's qubit, or in other words, changing it by an infinitesimal amount (what is the information of a bit is to that of a qubit).

Now, Bob receives the same qubit instantly as in the former case, however, somehow delving into Alice's new interpretation to represent both conjugate quantities as entangled. So, the transmitted quantum information is complete, no missing bit of classical information. What is different to Bob after penetrating into Alice's newly introduced intention is only to realize the obtained signal as gravitational: it turns out to be exhaustive to Alice's complete state reinterpreting it to be gravitational correspondingly measuring it by a relevant gravitational experiment. Then, Bob confessing Einstein's general relativity not allowing for any superluminal gravitational interaction is free to think of it to be an interaction due to "dark matter" and "dark energy": moreover, he can admit that all the prevailing "ocean of dark matter and dark energy" originates from a medium of "Allices" in any point of spacetime correctly messaging her own state absolutely completely and obeying the theorems of the absence of hidden variables in quantum mechanics (Kochen, Specker 1967; Neumann 1932). If Bob is not only a physicist, but furthermore both mathematician and philosopher, he can also admit that the ocean of countless "Alices" creates his reality by gravitation, in fact, being the total effect of all messages, each of which a certain Alice in a spacetime transmits in order to message her state. Bob might read Heidegger who wrote that we are a conversation, or that the being is a dialog or a language.

The present thought experiment suggests that two experimenters, possibly "Alice" and "Bob", exchange whether classical information by light signals or quantum information by qubits, and the one of them is macroscopic, describing the corresponding experience in terms of special and general relativity, but the other "colleague" is microscopic, quantum, and commensurable with the Plank constant (respectively with the Planck length, time interval or mass). The quantum researcher utilizes only quantum mechanics including the theory of entanglement and quantum information, but the other investigator, being furthermore macroscopic, uses only special and

general relativity, and classical mechanics. Nonetheless, either of both can respond meaningfully after receiving the colleague's signals, however translating into his or her conceptual framework. The conclusion is that the fundamental theories are absolutely equivalent to each other distinguishable only by its relevant language and quite different from the other one: the language of relativity, both special and general, can be described to be a local and "Lagrangian" language just that of classical mechanics; on the contrary, that of quantum mechanics is a nonlocal and "Hamiltonian" one therefore allowing for the stigmatized by Einstein "spooky" actions at a distance, but legitimated to be the real phenomena of entanglement and quantum information. Nonetheless, all of them can be represented absolutely equivalently also locally and in "Lagrangian" (language), but as gravitational phenomena.

Even more, each of both researchers can imagine the alternative picture and language of the world confessed by the other colleague absolutely correctly, though in an only mental way and contradict to her or his empirical experience correspondingly either quantum or human. Alice is able to restore Bob's picture and language of the world only substituting "Hamiltonian" language by "Lagrangian" after which the phenomena of entanglement will be transformed into those of gravitation. Alice's proper quantum language will be in turn restored by Bob by the "Standard model" able to describe all everyday whimsical adventures of Alice being quite natural in her quantum world, now, in terms of Bob's inherent macroscopic experience and the readings of his apparatus obeying smooth classical mechanics.

The next problem is: what about the cosmogony and cosmology of the universe in either of Alice's scientific worldview versus that of Bob. Since all of us live in Bob's macroscopic world, his cosmogony and cosmology would rely on the "Big Bang", a singular spacetime beginning of the universe, after which it has expanded and continues to expand now under all universal physical laws including energy conservation though the singular point at issue is a monstrous violation of all of them. Sharing Bob's understanding of the universe, another macroscopic researcher can anyway suggest that all anomalies violating the ostensibly universal physical laws are unfoundedly removed from the entire spacetime universe after the "Big Bang", but collected in the "Big Bang" itself and its "singularity". The translation of Bob's cosmogony and cosmology to that of Alice can rely on the understanding of the singularity of the Big Bang as a single special, and privileged reference frame, in the beginning of which the Big Bang has ostensibly taken place, at least in Bob's opinion. Following the language of relativity, being usual for Bob, he supplies that reference frame with a relative position, a relative velocity, and a relative acceleration, but he cannot determine to which other reference frame they are, namely, "relative". Anyway, he may admit two hypotheses, eventually equivalent to each other, about that problematic "relativity":

The one grants the reference frame of the "Big Bang" to be absolute since general relativity is consistent with the existence of some absolute reference frame, even in Einstein's opinion (Einstein 1920). The other one considers the class of all possible spacetime reference frames after the Big Bang, to the class of which the position, velocity, and acceleration of the reference frame linked to the Big Bang can be considered as relative. Anyway, Bob privileges a certain reference frame in both cases though in different ways: (1) by postulating for one to be absolute; (2) by considering

to be relative, but to the class of all possible reference frames in all the universe after the Big Bang. Delving into Bob's reflections about the alleged beginning of the universe, Alice might reduce them into their essence consisting in the fact that a certain reference frame has been privileged furthermore ignoring absolutely the way in which Bob justifies that privilege. Next, she has to translate the concept of reference frame from Bob's inherent "Lagrangian" language onto her own one, which is Hamiltonian, therefore needing to translate the concepts of velocity and acceleration defined correspondingly as the first and second, time derivatives from the variable of position: thus, being even defined inherently in "Lagrangian" (language).

Then, the position of the reference frame at issue is to be represented by four spacetime coordinates, the velocity would add still four spacetime coordinates, totally eight ones, and finally the acceleration needs still one speed to be defined: this means that sixteen spacetime coordinates are necessary and sufficient to describe exhaustively any arbitrarily accelerated reference frame in "Hamiltonian" (language). The analogy to the sixteen most fundamental particles of the Standard model (excluding the Higgs boson) is obvious. However, is it not occasional or superficial? One can immediately notice that the three fundamental interactions (electromagnetic, weak, and strong) correspond exactly, to the position (four coordinates or particles), the velocity (still four coordinates or particles), and the acceleration (still eight coordinates or particles) of some unknown, but anyway somehow chosen to be privileged reference frame. The Higgs mechanism (respectively the Higgs boson) may be also explained in the same conceptual framework:

Any mass at rest admits and needs a certain velocity strictly less than the speed of light in vacuum. So, the introduction of that speed implies some corresponding mass at rest, which also can be represented in "Hamiltonian" (language) as the Higgs mechanism of the spontaneous violation of symmetry. That symmetry turns out to be that of the light cone as to the initial "text" in (Lagrangian) language in which Einstein's theory of relativity is written. If one chooses an arbitrary point strictly within the light cone, its symmetry is broken, but that choice is inevitable for any nonzero and finite mass at rest. So, whatever nonzero finite mass at rest and the broken symmetry of the light cone are equivalent. Furthermore, the introduction of still one velocity for the acceleration repeats literally the Higgs break of the symmetry of the light cone since both velocity and acceleration are to be related to the point mass therefore identifying the violation of the symmetry of the light cone whether to the velocity or to the acceleration of the researched reference frame though yet unknown which it should be.

So, the Higgs mechanism describes the entire class of all possible breaks of the symmetry of the light cone due to any nonzero finite mass at rest in "Hamiltonian" language or in other words, this is the relevant translation from "Lagrangian" (language) being inherent for the concept of reference frame in special or general relativity. The sixteen spacetime coordinates are able to determine a certain accelerated reference frame, but not its mass at rest, only establishing for it to be necessarily non-zero and finite. The 17th particle of the Standard model, the Higgs boson, serves to determine unambiguously the exact mass at rest of the privileged reference frame at issue.

Furthermore, any of the 16 most fundamental elementary particles meant in the Standard model is featured by a basic wave function able to represent exhaustively all of its fundamental parametric

values. Those wave functions can be correspondingly and unambiguously represented as 16 qubits. Any qubit (meaning a nonempty qubit, but with a certain value "recorded" in it) is isomorphic to a unit ball in Euclidean space, within which a certain point is chosen, i.e., a spacetime position in the light cone in the final analysis. In other words, the 16 basic wave functions of the 16 most fundamental particles of the Standard model by the mediation of the corresponding 16 qubits determines unambiguously a certain accelerated reference frame, but in "Hamiltonian" language, being "non-native" for the concept of reference frame.

If any reference frame is considered in dynamics, it should be supplied with still one parameter, which is scalar (unlike the spacetime vector of position, velocity, and acceleration), and this is the mass at rest of the reference frame in question. The 17th particle of the Standard model, namely the "Higgs boson" represents that parameter in the proper discrete language of quantum mechanics, which is "Hamiltonian" as a relevant leap from zero mass to a certain non-zero and finite mass at rest, and thus by means of still one basic wave function expressible also by a relevant qubit.

In other words and summarizing, one can reasonably suggests that the Standard model represents in "Hamiltonian" (respectively translating from "Lagrangian" in "Hamiltonian" language) every single one of the parameters of a certain privileged reference frame possessing a non-zero finite mass at rest as the absolute reference frame, to which all moving reference frames can be determined relatively and unambiguously. Furthermore, each of them can be equivalently represented in "Hamiltonian" as if consisting of an arbitrary combination ("superposition") of the 17 basic wave functions of the Standard model so that one can trace back the way in which any quantum entity consists of those 17 most fundamental elementary particles.

However, the same picture of the world can be equivalently represented as a certain force or interaction influencing on any given reference frame and changing or not its relative motion to the absolute reference frame. That universal force or interaction is gravitation: at that gravitation as it is exhaustively described by Einstein's theory. Then, Einstein's gravity cannot be "added" relevantly (as "quantum gravity") to the other three, proper quantum interactions (electromagnetic, weak, and strong) since it only represents the same as those three forces but in another language, called here "Lagrangian" and being inherent not only for special and general relativity, but also for classical mechanics.

One can use the metaphor of the translation of any text from a human language into another, for example from Bulgarian into English or vice versa. One can admit an absolute exact, "transparent", ideal translation not adding or neglecting any meaning or sense (connotation): i.e., being the same to the original text after translating. Then, the translated text does not really do anything to the initial text, and the translation cannot complement the reality described by the original text in whatever way. Just this is the sketched above idea about the link between the Standard model and general relativity, after which quantum gravity is to be identified with entanglement at a proper quantum level or locally, but Einstein's gravity and the Standard model are to be identified as correspondingly the global and local aspects of the same utilizing either

"Lagrangian" or "Hamiltonian", but inherently and fundamentally describing the same physical reality in two alternatives, but equivalent ways.

Then, all attempts during the last about 100 years since general relativity has been created for gravitational interaction to be relevantly described as a still one along with the other three interactions in the Standard are so fundamentally in vain, as the efforts for constructing perpetuum mobile and forced the French academy at last to stop considering any projects of it. Anyway, the impossibility is different in those two cases but essentially linked though paradoxically therefore justifying anyway the metaphor at issue.

Indeed, the principles of thermodynamics being natural laws forbid any "eternal motion" implementable in *perpetuum mobile*. On the contrary, no natural laws prohibit quantum gravitation and it really exists as quantum information and accordingly studyable in all phenomena of entanglement, quantum teleportation, *etc*. Nonetheless it is impossible to be added to the Standard model being equivalent to it. Spoken otherwise, if any quantum system is considered as a single whole, just the Standard model is the theory relevant to it; however, if it is meant to consist of parts, quantum subsystems, all phenomena of entanglement appear inevitably, and therefore quantum gravity.

Summarizing, the fact that either the Standard model or the entanglement theory of gravitation is relevant depends only on the viewpoint to it rather than on physical reality by itself. This is the reason for gravitation not to be addable at least in the same way of a conservative generalization as electroweak interaction relates to electromagnetic interaction. Nonetheless, electroweak interaction refers to both electromagnetic and weak interactions just as gravitational interaction (after general relativity and the entanglement theory of quantum gravitation) does to the Standard model. Speaking loosely, one can say that gravitational interaction is "strong-electroweak".

VI. MORE REFLECTIONS ABOUT WHAT GRAVITATION IS

The paradoxicality of the metaphor about the hypotheses about quantum gravity by means of the impossibility of *perpetuum mobile* consists in the following. The impossibility of the *perpetuum mobile* (meaning Einstein's equivalence of energy and mass) prohibits any "creation ex nihilo". On the contrary, the equivalence of gravitation by mediation of entanglement and quantum information to the Standard model justifies directly that creation *ex nihilo* as an immediate corollary:

Pauli's particle paradigm in classical quantum mechanics embedded and culminated also in the Standard model does not implies yet that creation *ex nihilo*. On the contrary, it follows the standard scientific worldview of its impossibility particularly proclaimed by the French academy in relation only to *perpetuum mobile*. In fact, quantum mechanics can be interpreted to be consistent to both polar viewpoints equally well: to Pauli's particle paradigm, on the one hand, but not worse to its historical opponent the Bohr – Kramer – Slaters (BKS) theory[26] admitting the violation of energy conservation for Heisenberg's uncertainty (so-called fourth uncertainty of energy and time). However, the originally BKS theory by itself did not involve entanglement and

[26] Bohr, Kramers, Slater (1924).

the option for the eventual violation of energy conservation was not relevant in scientific debate between Bohr and Pauli. The latter's position was formally the correct one to the explicit formulations of the discussed radiation problem.

Nonetheless, Pauli's particle paradigm, especially after his theoretic, "paper" prediction of neutrino on the basis of energy conservation in quantum mechanics, became dominating far beyond the proper boundaries in which it was initially formulated. So, any violation of energy conservation was rejected and alternatively explained by some relevant new particle even so shortly living that it could be suggested only theoretically as being a virtual particle. Pauli's particle paradigm was supported, in fact only visualized, by Feynman's diagrams, after which one could be "convinced" that all elementary particles transform into each other only looking at the diagrams.

A problem only was that the number of those "elementary particles" grew fast reaching a few hundreds, even a few thousands therefore excluding for them to be really "elementary' being so numerous. The Standard model though without gravitation decided it reducing the number of the really fundamental elementary particles to 17. Nonetheless, the position of the present and other papers is for the Standard model to be reinterpreted as a semi-empirically established set of "epicycles" as if allowing for the "geocentric system" to be conserved rather than replaced by any newly introduced "heliocentric system"

The metaphor or analogy to the historical precedent of Copernicus's innovation can be described in detail as follows. One can imagine a counterfactual course of history, in which a sufficiently complete set of correcting epicycles had been involved including those for the motion of the Solar system through the Milky way or even through the universe, so that the predictions of the geocentric system were much more precise than those offered by its rival, the heliocentric one. Even more, those correcting epicycles might be systematized and reduced to a certain number, for example, only "17" ones by a corresponding relevant "Standard model" of all epicycles and supported of a dogmatic organization of science relied on the "blind reviews" of "normal science" (after Thomas Kuhn), therefore stigmatized all articles confessing the heliocentric system: no need of pyres since the geocentric system, furthermore consistent with the authority of the "Holy Church" would be established forever.

As this is very well known, the suggested counterfactual course of history was not realized, and Copernicus's system took place as well as Giordano Bruno's pyre (in fact, quite in vain and useless). However, it might be realized now, following the sketched metaphor or analogy. The change from the geocentric system to the heliocentric system would correspond the change from Pauli's particle paradigm or the Standard model both excluding whatever creation *ex nihilo* as antiscientific to those worldviews advocated in the present and other papers.

Neither the step to Einstein's gravitation (in the way in which it is standardly interpreted) nor that to the eventual entanglement theory of gravitation imply that creation *ex nihilo* though both admit the violation of energy conservation: the former due to the conservation of energy-momentum inferred by Hilbert or Einstein, and the latter because of involving non-Hermitian operators implying non-unitarity. Anyway, the crucial step is that suggested in the present paper and consisting in the "dialectical" reinterpretation of the equivalence of Lobachevsky's approach

and that of Riemann to non-Euclidean geometry, which can be also realized as a more radical penetration into the logical origin of Einstein's general relativity also interpretable as quantum gravity by the mediation of entanglement and quantum information. The ontological contribution of Hegel's speculative dialectic is essential though it is seen to be "ontomathematical" in the present context implying that creation *ex nihilo* by virtue of ontomathematics, revealing the proper mathematical roots of a so fundamental physical force or interaction as gravitation[27].

One can again pay attention to the aforementioned problem of whether the transformation of the universe from invisible into visible after the propagation of light in vacuum changes some physical laws or rather not. The short answer is "Not at all!" according to the consideration above, which may be summarized in a few statements. Really, any different reference frames accessible in general relativity suggest different translations in "Hamiltonian" (language) and thus different "Standard models", each of which is relevant to a certain reference frame. Speaking loosely, the change of the viewpoint from a reference frame to others changes also the set of the most fundamental elementary particles as those in the "Standard model" obeying energy conservation as well as Pauli's paradigm. Consequently, the Standard model established on our planet will be valid in an infinitesimally small neighborhood about it since the corresponding 17 most fundamental particle would be different far enough.

However, the infinitesimal neighborhood should be identified with the visible universe rather than with an infinitesimal neighborhood about a certain spacetime terrestrial point. Again, or speaking loosely, one can say that all spacetime in definition coinciding with the visible universe represents (or transforms) the infinitesimal neighborhood in which the "terrestrial Standard model" is properly valid in a finite area such as the visible universe or spacetime. That transformation is well known in differential calculus and even trivial there, consisting in the substitution of any variable in any infinitesimally small vicinity of its value ("point") by its first derivative.

Indeed, the fundamental constant of the speed of light in vacuum determining the expanding boundaries of the visible universe replaces the variable of spacetime position, in which the Standard model is properly valid in an infinitesimally small neighborhood, with the finite domain of the first derivative of velocity. In other words, the Standard model is slightly different, for

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[27] The present paper, introducing the fundamental and philosophical conception of "ontomathematics", allows for Newton's original philosophy to be reinterpreted as "naïve ontomathematics" since the Cartesian abyss from mathematics to physics was not commonly accepted as it was later, incl. in Einstein's age or works. Many papers discuss Newton's proper more or less implicit philosophy and called often "Newtonianism" (for example: Kasz 2016; Belkind 2013; Henry 2013; Janiak 2013; 2012; 2008; Watkins 2013; Machamer, Mcguire, Kochiras 2012; Galluzi 2010; Grant 2010; Ducheyne 2009; 2006; 2006a; 2005b; McGuire 2007; Grabiner 2004; Force 2004; Mandelbrote 2004; Osler 2004; Shapiro 2004; Stewart 2004; Young 2004; McMullin 2001; Stinner 2000; Guicciardini 1999; 1993; Albert 1997; Borzeszkowski 1993; Bonsiepen 1993; Buchdahl 1993; Garrison 1993; Gjertsen 1993; Graneau, Graneau 1993; Guicciardini 1993; 1987; Ihmig 1993; 1993a; Kluit 1993; Priest 1993; Vigier 1993; Wahsnerin 1993; Werle 1993; Wolf-Gazo 1993; Johnson, Chandrasekar 1990; 1990a; Laing, Jones 1985; Gabbey 1980; Cohen 1978; Forbes 1978; Westfall 1962; More 1943; Metzdorf 1942). The concept of ontomathematics is also available till now (though partly or implicitly) as the problem of "mathematization", including as a philosophical one; for example, in Lenhard, Otte 2018 López-Gay, Sáez, Torregrosa 2015; Massimi 2010; Roux 2010; Trelinski 1983; Wheeler 1982, Zahar 1980.

example, in Sofia and Princeton, as two different points in Euclidean space. Nonetheless, it is the same if two researchers in the two cities accordingly, exchange light signals about experiments which each of them accomplishes in the own localization for confirming or rejecting the Standard model since the *light*-signal exchange itself has been substituted in advance the variable of Euclidean position by its first derivative of speed obeying the light barrier. If one translates the same observation into mathematics (from physics), an expansion of any infinitesimally small vicinity about any point of any vector space would correspond. So, if one links a local observer in any point of that vector space, she or he would watch the expansion of the global vector space just as a terrestrial astronomer such as Hubble himself establishes that the universe expands.

The relevant conclusion is that the universe shares a universal property in any vector space, or reflecting philosophically that the expansion of the universe is a mathematical law rather than a physical one as the commonly accepted, ostensibly scientific prejudice states. One can visualize the mathematical (rather than physical) law by Einstein's visit to Hubble's observatory, after which the former proclaimed to be his "biggest blunt" (at least according to Georg Gamow[28]) for the additionally introduced "cosmological constant" in the equation (nowadays called the "Einstein field equation") necessary for the universe to be kept "stationary" (contradicting Hubble's astronomical observations), on the one hand, but obeying "Mach's principle" (Einstein 1918) alleging that mass and energy are all possible sources of gravitation.

Though Einstein himself introduced "Mach's principle" and the additive member, counting the cosmological constant, in the field equation of general relativity just for the universe not to be expanding, the real contemporary situation is rather confused and quite inconsistent to his initial intention, by the by, reversely repaired by the proclamation of his "biggest blunt". The expansion of the universe is commonly accepted and frequently, but complemented rather paradoxically by the cosmological constant together and the additional member in the Einstein field equation. However, the relation to "Mach's principle" is especially ridiculous and even absurd:

The huge amounts of "dark matter" and "dark energy" in the universe, furthermore observed only by virtue of their gravitational effects are rather consistent to the suggestion that there exist other sources of gravitation than those postulated by "Mach's principle", namely mass and energy, but in fact implicitly meant to be "light mass" and "light energy" in the context of general relativity. Nonetheless, any violations of energy conservation in physics at all and Pauli's "particle paradigm" in quantum mechanics are rejected though the 2022 Nobel Prize for entanglement and quantum information implicitly implies those. Contemporary physics is "pregnant" with a new scientific revolution, which will generate most probably relevant, also revolutionary social changes[29].

The present paper advocates an even more radical interpretation of the rejection of "Mach's principle", after which not only other source of gravitation exists, being furthermore crucially prevailing in the universe according to all relevant astronomical observations, but that source is "ontomathematical" and thus contradicting the Cartesian organization of cognition and society in

[28] Gamow 1970: 44.
[29] In more detail in: Penchev 2023 March 13.

Modernity. Then, the corresponding ontomathematical interpretation of the Einstein field equation and the "fermentations" about it are the following:

The expansion of the universe is due to ontomathematical reasons in the final analysis. The cosmological constant (eventually generalized to be a "cosmological variable") and its member are to be added in the Einstein field equation also following ontomathematical causes rather than for the universe to be kept "stationary" as Einstein's initial intention was more than a century ago. Mach's principle is to be absolutely rejected: at that allowing for the ontomathematical sources of gravitation (such as entanglement and quantum information) to be proclaimed as prevailing and identified with dark matter and dark energy.

Then, those confusions or "fermentations" about the Einstein field equation can be explained in a thoroughly new way, namely as troubles about the global ontomathematical worldview tending to unify philosophy, physics, and mathematics therefore revolutionizing each of them in particular. Surely, this means an absolutely different worldview inevitably accompanied by too much misunderstanding and resistance in the course of its establishment.

One can now elucidate why the propagation of light in a vacuum does not change any physical law permanently transforming the invisible part of the universe into visible one. The Standard model though in an improper, "Hamiltonian" language describes an absolute reference frame being equally relevant to both visible and invisible parts of the universe. So, the invisible part is a part just of our own universe rather than of some other universe. However, that statement though being intuitively quite justified can be rigorously proved only after granting the afore-sketched ontomathematical scientific picture of the world, for example, as follows:

Nonlocal interactions by entanglement preestablish (as if recollecting Leibniz's "preestablished harmony") just a single certain reference frame as being the same and thus absolute to all the universe. Those nonlocal interactions are equivalently mapped as that prevailing gravitation originating from the dark mass and dark energy, to which the insignificant gravitational influence (approximately 20 times weaker) of the light part of the universe is added. Then, the universe as a whole can be represented as a single reference frame to which any part of it is in a corresponding state of relative motion. That nonlocally preestablished reference frame at issue is the proper *absolute* reference frame valid to all the universe as being the same.

The absolute reference frame of the universe is inherently expressed in "Lagrangian" (language), but it can be translated in "Hamiltonian" as above: that is as the Standard model valid as a universal physical law to all the universe. By virtue of that, the transformation of the universe from invisible into visible does not change the Standard model being valid to any point or reference frame in the universe regardless of whether it is visible (for example, for any terrestrial astronomer) or not. However, that universal validity of the Standard model cannot be proved otherwise that by the nonlocally preestablished absolute reference frame of the universe. This means that the universal Standard model in turn implies the pre-establishment of a single reference frame as the same in relation to both visible and invisible parts of the universe and thus absolute to all the universe *only nonlocally*.

In fact, the hypothesis of the Big Bang does not follow from the existence of an absolute reference frame being identical to any part of the universe though the reverse statement (that the Big Bang privileges a certain reference frame ostensibly taking place just in it, also frequently notated as the singular point, or the point of the initial singularity) is correct. The difference can be visually demonstrated by any events preceding the Big Bang since a spacetime reference frame though privileged to be absolute admits anyway physical facts determined by a negative temporal coordinate and thus happened before the beginning of the reference frame.

The privilege of the absolute reference can be crucially distinguished from as if linkable or originating from the Big Bang as follows. One can imagine a "field of the Big Bang": that is as an omnipresent and omnitemporal spacetime medium, in each point of which a tiny "Big Bang" takes place after the conjecture of the violation of energy conservation: the creation *ex nihilo* occurs in any spacetime point of the universe. However, the dominating scientific worldview stigmatizes those medium, in which creation *ex nihilo* is fundamentally possible as absolutely wrong even anti-scientific. Then, all violations of energy conservation are merely "summoned" in the singularity of the Big Bang, and each spacetime point of the universe might obey energy conservation or Pauli's particle paradigm (at least until the discovery of "dark matter" and "dark energy" as well as the establishment of entanglement and quantum information by the 2022 Nobel Prize in physics).

The aforementioned field of omnipresent and omnitemporal "tiny Big Bangs" can be also interpreted as a universal spacetime medium of decoherence or decoherent processes, by which the degree of entanglement decreases, and the nonlocal dark phase of the universe transforms gradually in its "light phase" usually identified with the subject of physics until now. Then, the singularity of the Big Bang (as in the usual "scientific" representation) should be a mythical rather than real event and due to the Cartesian "spectacles" by which contemporary science and particularly physics sees the world: in fact and rather paradoxically, quite anti-scientifically.

The cause of that "scientific anti-scientificity" is the ostensibly "scientific" dogma forbidding any creation *ex nihilo*, any violation of energy conservation and heralding Pauli's "particle paradigm" in quantum mechanics culminated in the Standard model. The Standard model is true, but not "all the truth", and only its "local, visible, and light part", out of which is the crucial part of the universe turning out to be mainly "nonlocal, invisible, and dark", gravitation, entanglement and quantum information. All of them can be likened to Kelvin's famous "little clouds om the horizon of physics" in the eve of the 20^{th} century: however now paraphrased in relation to the horizon of physics in the eve of the 21^{th} century, and already transforming in forthcoming revolutionary "scientific thunderstorms".

The beginning of the universe has not originated from the mythical and semi-religious "Big Bang", but from mathematics alone, speaking loosely, "ex nihilo" and even permanently in any spacetime point and following necessary mathematical laws. The light and visible part of the universe appears from its nonlocal, invisible dark phase everywhere and always, in each microscopic and quantum event of decoherence therefore violating all dogmas of energy

conservation really valid to the tiny “light phase of the universe” fundamentally not being able to explain the genesis of the universe and generating the myth of the Big Bang instead of truth.

The social reason for that afore-sketched paradoxical “antiscientific scientificity of science” is due to the fact of being too connected to society. The allusion to Copernicus’s revolution (by the by, an allusion already utilized by Kant himself in order to qualify his innovative transcendental approach in philosophy) is quite suitable for the description of that too strong link between science and society and preventing scientific revolutions as generating “blood” (really or metaphorically) social revolutions in the final analysis.

Indeed, on could trace back the French revolution from Copernicus’s abstract theory, at least at first glance, quite peaceful and “ingenuous”. Society resists always to any revolutionary transformations in humankind’s worldview since it and the corresponding social order and hierarchy being absolutely necessary for its normal functionating rely on that worldview, and as to the particular case after the “Copernican allusion”, on the geocentric system confessed by Christian religion and Church as an absolute dogma and relying on Ptolemy’s or Aristotle’s scientific authorities as well.

Nonetheless, the heliocentric system contradicting the geocentric system is scientifically true. In fact, Copernicus’s new theory though consistent with some ancient suggestions was not interpreted socially, as relating directly or indirectly to the established social order and hierarchy, in virtue of which Copernicus, furthermore himself belonging to the clerical hierarchy (being a canon), was not persecuted during his life (including because of his uncle’s protection who was Prince-Bishop of Warmia). The cause consisted in the fact of the then too slow communications between different subsystems of society for realizing the potential danger of the heliocentric system to the Church, social order and hierarchy resulted in a few centuries later in social revolutions such as the French one.

One may try to forecast the future revolutionary influence of the present fundamental discoveries in physics changing the entire organization of cognition in modern science, after which philosophical ontology is replaced or generalized to ontomathematics and the realization of gravitation as the corresponding onto-mathematical “force” able to create realty rather than as still one physical interaction along with those three meant by the Standard model. The present paper is not targeted in the complete description of the eventual social sequences in detail, but to sketch them cursorily only emphasizing that they would be essential and considerable. A new and more general (than energy conservation) natural law about quantum information conservation would regulate all processes of “creating ex nihilo”, or conversely, the transformation of mass and energy into “pure” quantum information, therefore allowing for a new horizon of optional technical implementations far beyond the even remote in the future boundaries of today’s technics relied on the corpus of all possible contemporary scientific cognition.

However, not the technical sequences would be crucial for society compared with vanishing any social hierarchy and weakening relevantly the social order to a fluid state of free creativity, archetype of which might be the Internet network now with variety of all links very quickly changing each second. Of course, the world web does not have any “boss”, “deputy bosses”, etc.,

to whom the users of the network would be the lowest level. This even sounds as a joke. Any hierarchy might be likened to a solid “crystal”, absolutely impossible in the fluid medium of the Internet now, moreover in the future and gradually absorbing all the society obeying maximal creativity.

The new law of quantum-information conservation (eventually replacing that of energy conservation as the most fundamental principle of nature) implies the future fluid state of society at issue absolutely unlike that of energy conservation. Indeed, energy conservation needs and requires an unambiguous hierarchy of all energies so that the influence of the physical interactions on any system is inversely proportional to its energy. Speaking loosely, much energy for a physical system means that it is a “boss”: it influences strongly on the others in the interactions, but the converse influence is inconsiderable. Much energy for a physical system allows for it to dominate.

However, that energetic hierarchy features only the local, visible and light part of the universe, being an immaterial part of it, only in the framework of which energy conservation is properly valid. The universe as whole obeys in general to an “anarchy” rather than to any hierarchy so that an electron can be equal to a star regardless of the fact that their energies differ from each other by decades of exponents. The electron at issue is energetically “subordinated” to the star only as to the local and visible part of the universe, for which the metaphor for the “screen” of the universe by itself and as a whole can be utilized just as the screen of a computer is not the most essential part though being probably the most noticeable one and immediately “striking”.

However, what is on the screen of a computer, “locally” and “visibly” is a result, in fact, due to calculations being processed “nonlocally” and “invisibly” in its “dark part” as well as the causality or hierarchy of what is pictured on the screen, for example, such as a movie only projected on the screen, is absolutely irrelevant to the real computational reasons causing the motions of what is visible on the screen. Of course, this is only a metaphor representing partly and inessentially the relation between the light and dark parts of the universe. Anyway, the hierarchy of what can be watched on the screen is absolutely immaterial to the calculative medium really produced the moving images on the screen, and the figure tends to emphasize that.

Particularly, the organization of modern science obeying the Cartesian episteme and hierarchy (after Thomas Kuhn’s “normal science”) dominated by a single paradigm would be irrelevant to that dark, fluid, and crucially prevailing phase of the future society if it would correspond to the worldview about the dark and nonlocal part of the universe really ruling what is visible on the local “screen” of the universe. Furthermore, the “sites” trough which one can “surfs” now utilizing the “quantum computer” of reality (i.e., that of the universe) are reliably divided in different sciences or disciplines excluding any direct contradiction for obeying inconsistent or incompatible hierarchies since each site introduces a single hierarchy, that of the dominating paradigm in the disciplines at issue and needing a relevant “scientific revolution” (again after Kuhn) to be changed by a new hierarchy imposed by a new revolutionary scientific theory.

However, the worldview advocated by the present paper alleges gravitation (including the nonlocal entanglement theory of it) to be that force behind the stages of different sites of reality creating itself and indistinguishably linked to the non-energetic and non-material processing of

quantum information in "dark depth" of the universe and only represented as that surficial reality, "ready for" all human local experience, experiments, or observations exhausting any possible and admissible basis of classical science.

The next paragraphs will try to penetrate and reinterpret the sketched above briefly stages of the theoretical history of gravitation from the advocated ontomathematical viewpoint to it.

VII. FROM NEWTON'S GRAVITY TO EINSTEIN'S GRAVITY

Newton created the first *mathematical* theory of *universal* gravitation, therefore revolutionarily substituting Aristotle's qualitative physics not able to predict whatever exactly, as this would be required for any physical theory and generalized, for example, by Popper as the line of demarcation to metaphysics. Meaning the viewpoint of the present paper, the crucial step done by Newton is to be reinterpreted as that from Aristotle's ontology, relied on the logic of propositions invented or at least articulated explicitly by him, to ontomathematics, which can be legitimately called Newton's ontomathematics since he confessed just that understanding of it (due to which, in particular, he titled his work on universal gravitation: "Philosophiæ Naturalis Principia Mathematica", i.e., "The Mathematical Principles of Natural Philosophy") therefore sharing the same philosophical ontomathematical worldview to gravitation as the present paper.

However, his not only intuitive insight, but furthermore and first of all, detailed elaborated approach turned out to be inaccessible to the next generations enumerated his theory of gravitation among physics more and more gapped from mathematics and philosophy after the Cartesian spectacles to reality, predetermining for it to be divided by an abyss extending from the empirical "body" studied experimentally by physics, on the one shore of it, to the only mentally accessible "mind" of mathematics, logic and philosophy, on the opposite shore. Newton lived in Descartes's age, in which the Cartesian organization of cognition in Modernity was not established at all, however, all future readings of Newton's theory of gravitation obeyed it more and more as Cartesianism became gradually dominating, after which *Newton's discovery of ontomathematics* (in fact, only illustrated by his properly physical theory of gravitation) was irrevocably lost[30].

Newton's main contributions in both physics and mathematics are (commonly accepted): (1) the "method of fluxions", i.e., infinitesimal (or differential and integral) calculus; (2) his mathematical theory of universal gravitation; (3) mechanics inferred from his famous "three principles"[31]. Nonetheless, they are not usually considered to be united, but rather enumerated in

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[30] As, by the way, Fermat's original proof of his last theorem claimed by himself, but not written: "Hanc marginis exiguitas non caperet" (Fermat 1670: 338-339).

[31] The definition of those contributions to be "main" is more or less conventional obeying the specific consideration in the present paper. There exist many enough articles, studies or books discussing different aspects or parts of Newton's heredity, for example: Fox 2016; Kvasz 2016; Palenik 2014; Belkind 2013; Schuster 2011; Darigol 2010; McGuire 2007; Sellés 2006; Iliffe 2004; Reyes 2004; Ramati 2001; Guicciardini 1999; Lamb 1994; Bonsiepen 1993; Graneau, Graneau 1993; Neuser 1993; Snobelen 1998; Garrison 1987; Aoki 1996; 1992; Greenberg 1996; Moore 1993; Moretto 1993a; Pater 1993; Petry 1993; 1993a; Sarlemijn 1993; Steinle 1993; 1993a; Muraskin 1992; Meli 1991; Whitrow 1989; Hojman, Hojman 1985; Cushing 1982; Cohen 1978; 1964; Ramakrishnan 1973; Ferguson 1968; Wisdom 1941; Mordel 1927; Snow, Sugden 1924;

mathematics and physics correspondingly and thus even opposed as the organization of modern cognition needs and requires. The latter two contributions, though undoubtedly both belonging to physics, also are not interpreted together, but rather independently of each other, at least until Einstein's general relativity:

Indeed, his fundamental principle underlain the foundations of that theory: namely, about the exact equation of gravitational and "inertial" (i.e., determined by Newton's second principle) masses[32] (for example, visualized by Einstein himself with the famous thought experiment about an observer in an accelerated elevator due to some unknown force causing the acceleration of the elevator or a certain gravitational field resulting in the same observable effect in relation to the same mass) can be now reinterpreted in the present context as able to unify Newton's mechanics inferred from the three principles with his theory of universal gravitation and then to deduce Einstein's general relativity from that unification since it in turn implies the general principle of relativity postulating for all physical laws to be invariant to arbitrarily accelerated reference frames rather that only inertial ones according to the special principle of relativity[33].

The special principle of relativity also can be "destructed" (or "deconstructed") to Newton's first principle, after replacing the immovable ether and the absolute space and time due to the absolute reference frame linkable to it, with the newly introduced "light ether" because of the postulate of not exceeding the speed of light in a vacuum and particularly implying for the absolute (and also absolutely divided and quite different) space and time to be united in spacetime and Minkowski space. So, still Einstein's special relativity can be seen only to reinterpret or "repair" Newton after the substitution of the classical immovable ether with the light cone of Minkowski space, thoroughly identifiable also by its unique property for all points to be at a zero spacetime distance from each other. Speaking loosely, special relativity is: "Newton's mechanics with Einstein's "light ether".

In fact, general relativity only deepens or continues the repairment of Newton's "Mathematical principles of natural philosophy" proclaiming that universal gravitation and the mathematical mechanics inferable from the three principles is to be unified therefore implying the "universal covariance" of general relativity or the principle of relativity relevant to it. However, that "repairment" undertaken by Einstein is more necessary to himself and modern physics confessing Cartesianism and the corresponding organization of cognition rather than to Newton and his age yet living in "Eden", in which mathematics, physics, and philosophy were naively or

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[32] The "equivalence principle" meaning the equivalence of gravitational and inertial masses is widely discussed: Hetzroni 2020; Castaing 2018; Everett 2018; Fox 2016; Goto, Natti, Natti 2010; Kajari, Harshman, Rasel, Stenholm, Süßmann, Schleich 2010; Singh 2009; Drake 2006; Rabinowitz 2006; Mensky 2004; Kawai, Shibata, Tanaka 2000; Shirafuji, Nashed, Kobayashi 1996; Kluit 1993; Tsai 1986; Huang 1985; Gertsenshtein 1984; Börner, Schlieder 1980; Kolosnitsin, Myheev, Osipova, Stanyukovich 1975; Cohn Roll, Krotkov, Dicke 1964; Brown 1960; Pockman 1951; Thomas 1924; etc.

[33] That approach does not contradict the standard one about the relation of Newton's universal gravitation and Einstein's general relativity considering the latter as a conservative generalization of the former (e.g., Sim 2015; Aksirov 2009; Aoki 1996; 1992; Straus 1968).

inexperiencedly the same, and Newton could title his work as he did, being, however. absurd or nonsense as to Einstein or any other contemporary physician.

In other words, Newton's original intention unified universal gravitation and mathematical mechanics since the gap of them invented by Cartesianism though being in Newton's age a historical fact was not yet that absolutely obligatory imperative ruling and subordinating all modern cognition and particularly physics in Einstein's time or nowadays. So, one can suggest the following ontomathematical picture of the world relevant to Newton himself and inspired him for the enumerated above three main contributions: (1) the properly mathematical world to which differential and integral calculus refers allowing for infinitesimally small quantities to be investigated as finite and accessible to human experience; (2) the theory of universal gravitation describing the way for the mathematical world to be transformed into physical one; (3) the new mathematical mechanics deducible from the three principle thus able to describe the physical world by an inherently mathematical method.

So, the Cartesian gap being natural in Einstein's age but definitely not in that of Newton himself would oppose the contribution enumerated as (1) to that as (3), and the theory of universal gravitation is the bridge between them rejected thoroughly by Cartesianism divided them insurmountably by an abyss. So, gravitation in Einstein's epoch, though intended by Newton as a link, is reinterpreted to be absolutely on the shore of physics together with Newton's mechanics and its three principles, both requalified to be physical theories verifiable only by experiments and to which mathematics is only auxiliary suggesting tools by relevant models or as a "language".

So, Einstein as a son of his time created general relativity as a purely physical theory able to connect consistently Newton's universal gravitation and mechanics, in fact, reinterpreting them in the obligatory Cartesian manner, but so fremd to Newton's own original intention to suggest "The Mathematical Principles of Natural Philosophy". Then, one can try to restore general relativity in the initial and inherent framework of Newton's proper intention, after which Einstein's "general covariance" though being equivalent of the general principle of relativity (formulated in physical terms), can be returned to Newton's plan for gravitation to be a bridge between both physical and mathematical shores of the abyss heralded by Cartesianism, but not yet established during Newton's life.

Then, one can reinterpret both covariance and contravariance of general relativity in the proper Newtonian worldview: that is, their mismatch, conditioning the curvature of pseudo-Riemannian space and thus gravitation in the final analysis, can be situated correspondingly on the mathematical shore (for "covariance", but in fact conventionally) versus that of physics so that gravitation according to general relativity is the natural link over the Cartesian abyss absolutely expressing Newton's original design. Thus, Einstein's "general covariance" already restored in Newton's plan would insist that the world ostensibly only physical is actually both physical and mathematical since gravitation is a relation, representable by the relevant tensor calculus, of "mathematical" or "mental" covariance and "physical" or "bodily" (a.k.a. "empirical" and "experimental") contravariance.

Bracketing, one might notice that general covariance being an equivalent of the general principle of relativity and closely connected to the equality of gravitational and inertial masses implies that gravitation though being a single interaction should be equivalent to the three fundamental interactions meant by the Standard model. Indeed, following Newton's second principle, each of those three interactions would generate accelerations of the entities possessing a certain mass (respectively, also energy according to Einstein's special relativity) and might be summed totally: in turn, being equivalent to the effect of a certain gravitational field able to cause the same acceleration. So, Einstein's general relativity directly implies as a corollary that gravitation and the other three interactions are equivalent therefore rejecting the option for quantum gravity to be still one quantum interaction along with those three ones at issue, ostensibly the "weakest" among them and thus much, much weaker than the "weak interaction" of the Standard model.

Meaning the considerations above, one can suggest the conjecture that Einstein anyway (in an epoch in which the Cartesian episteme dominated and continues to dominate absolutely) restored Newton's ontomathematical project at least partly, in relation to gravity, however, as a *geometrical* physical theory. As this is discussed in detail above, geometry occurred initially as an empirical theory admitting experiments similar to today's physical experiments though too elementary in comparison with them. Euclid was who rewrote it as a mathematical theory simultaneously establishing a pattern for mathematical cognition universally valid even nowadays, known as deductive and axiomatic method. In a sense, thus Einstein accomplished a transition inverse to Euclid's contribution by creating a physical theory as a geometrical one, i.e., mathematical in the final analysis, at least after Euclid's revolution.

So, one may state that Einstein restored anyway Newton's ontomathematical plan about the mathematical principles of natural philosophy though in a "allegorical, Aesopian language" forced by the strict censorship imposed even as an unconscious auto-censorship by Cartesianism to science at all, including to physics in particular. So, Einstein's "physics as geometry" can be interpreted as an Aesopian allegory due to the Cartesian censorship[34] to Newton's ontomathematical design of gravitation. Then, the one shore of the Cartesian abyss corresponds (in an Aesopian manner) to vector covariance, and the other shore: to vector contravariance. So, general relativity is an ontomathematical theory exactly in Newton's testament but formally notated in an Aesopian theory of physics for avoiding persecutions by the Cartesian "inquisition", even unconsciously as an auto-censorship[35].

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[34] One may notice that Einstein main contributions (i.e. special and general relativity) consist in the translation of proper mathematical ideas of so great mathematicians as Poincaré ior Hilbert into the standard empirical and experimental language of physics therefore overcoming the "Cartesian censorship": cf.: Minguzzi 2011; Giné 2010; Aksirov 2009; Hacyan 2009; Tresoldi 2009; Gingras 2008; Stachel 2005; Darrigol 2004; 2000; Logunov, Mestvirishvili, Petrov 2004; Martínez 2004; Galison, Burnett 2003; Shima 2002; Rowe 2001; Corry 1998; Miller 1992; Earman, Glymour 1978; Giannoni 1970; Goldberg 1970; 1967; etc.

[35] Many enough papers discuss Einstein's philosophy being more or less implicit; for example: de Waal, ten Hagen 2020; Laudisa 2017; Wanas, Youssef, El Hanafy, Osman 2016; Agassi 2015; Rindler 2009; Galison, Burnett 2003; Aronov, Boi 1996; Wang 1995; Borzeszkowski, Treder 1993; Pakhomov 1986;

However, one can think of Einstein's general relativity as an only partial resurrection of Newton's ontomathematical project since it refers only to the latter two aforementioned contributions of Newton, but not touching the former one being furthermore properly mathematical (at least for scientific "common sense"): namely that of infinitesimal calculus. This is only seeming, though, because general relativity is a conservative generalization of special relativity and its postulate for not exceeding the speed of light in a vacuum where the straight lines of light propagation are generalized to arbitrary geodesic curves in pseudo-Riemannian space. In fact, one may easily demonstrate than infinitesimal locality (as in the "method of fluxions") is again meant in an Aesopian language of physical allegories rewriting special relativity geometrically, by means of Minkowski space:

Indeed, only the imaginary domain of Minkowski space is granted to make physical sense and thus physics is postulated to study only the light, visible and local part of the universe accessible furthermore to human empirical experience or experiments by the exchange of light signals. The quantity of velocity substitutes any infinitesimal neighborhood about each space point by its time first derivative being finite and thus accessible to experience and quantitative measurements unlike its infinitesimal space counterpart. So, Minkowski space and thus special relativity (hence general relativity) translates Newton's "fluxions" as "velocities" according to the imperative of the inevitable Aesopian language of physics, however, that translation is not literal since it enriches the input and proper mathematical text with the boundary of the speed of light in a vacuum, which be back reflected as the boundary of locality or infinitesimality in a purely mathematical sense[36]:

Meaning the idea of unifying physics and mathematics by ontomathematics in Newton's design of the "mathematical principles of natural philosophy", the fundamental concept in Einstein's relativity about the observers' reference frames can be reinterpreted by both local (infinitesimal) and global observers unambiguously distinguished by a boundary between them such as the exact constant of the speed of light in a vacuum as to our universe therefore generalizing it to be an arbitrary finite parameter able to divide any infinitesimal neighborhood, e.g., about each space point from any finite value of a first derivative as a function of all infinitesimal neighborhoods at issue, such as all velocities.

In other words, the newly introduced "mathematical observers" (one of them being local or "infinitesimal", but the other global just "Alice" and "Bob"), such as they establish the invariance of all mathematical and physical laws to both, would be forced to involve a parameter as a boundary between them to the class of which belongs the speed of light in a vacuum. Its definition would be dual to both observers, not causing any contradiction between them, though: (1) as to the local observers, all quantities such as space coordinates are infinitesimal, but their first derivatives such as time derivatives or velocities are finite; (2) as to global observers, all quantities such as

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Howard 1985; Montminy 1995; Paty 1995; Vigier 1993; 1988; Peres 1985; Zhou 1985; Feyerabend 1984; d'Espagnat 1983; LaLumia, LaLumia 1981; Pyenson 1980; Hanle 1979; Penrose 1979; Zahar 1977; Ballentine 1972; Holton 1968; Franquiz 1964; Nagel 1950; Frank 1949; McNabb 1925; Carr 1922.

[36] One may compare with: *Coleman, Korté 1995*.

space coordinates are (on the contrary) finite, but their first derivatives such as time derivatives or velocities are infinite (that is, infinitesimally great).

If both of them need the same picture of the world (this means all physical and mathematical laws to be the same regardless of whether the observers are local or global), they must to introduce the boundary between them (to the class of what boundaries the speed of light in a vacuum belongs being specified as to our own universe as a finite value) so that it is the least possible value among all first derivatives (such as velocities) as to all global observers, but simultaneously it is the greatest possible value among all first derivatives (such as velocities), after which both global and local observers can states that all quantities measured by each of them are finite and can be granted to be the same though complementary to each other since one has to establish in advance whether the experimenter who measures is local or global.

The lesson is the following. If one means the class of all those parameters of boundaries between the local and the global (respectively between the infinitesimal and the finite) rather than a certain value belonging to that class (which can be particularly specified to be the speed of light in a vacuum), that boundary can be introduced only and purely mathematically. This means after generalizing that one may distinguish what is mathematical from what is physical in an only and purely mathematical way, namely: any consideration to a class of equivalence is mathematical, but any consideration referring to a certain value among the former class is (on the contrary) physical.

Furthermore, the distinction, being so crucial for Cartesianism and for the omnipresent organization of cognition in Modernity, between the mathematical and the physical, i.e., whether a certain scientific subject is mathematical or physical is (in fact) absolutely conventional. Indeed, any proposition is equally valid being formulated to all the class therefore establishing for the class at issue to be a class of equivalence or being formulated to any element of that class in turn implying for the class at issue to be a class of equivalence as well.

The same option for the purely and only mathematical distinction of mathematics and physics within mathematics alone and thus suggesting to be interpreted as ontomathematics can be revealed also in the foundation of mathematics: by the distinction and equivalence of propositional logic as the universal “zero-order” logic of all mathematics, on the one hand, and set theory as the class of all possible first-order logics, i.e., all theories claiming to be mathematical, on the other hand. Regardless of that distinction, both can be considered mathematically to be the same Boolean algebra.

Then, one can notice that the discussed above option for ontomathematics, particularly in relation to the newly introduced invariance to both global and local observers, has been, in fact, inbuilt in the foundations of mathematics a long time ago, at least since Cantor’s set theory or Bourbaki’s project to infer all branches of mathematics on the basis of set theory, on the one hand, but it has generated a series of antinomies at least since Russell’s paradox as well Gödel’s dichotomy about the relation of arithmetic to set theory: either incompleteness or contradiction, on the other hand. In truth, arithmetic or the finiteness of all natural numbers by virtue of the axiom of induction can be interpreted as a universal model of locality in mathematics and thus opposed to set theory or to any actually infinite set by virtue of the axiom of infinity in turn realizing

globality in mathematics. The inherent incompleteness of locality to globality can be easily overcome following the design of quantum mechanics forced to reconcile both discrete (for finiteness in arithmetic) and continuous (for infinity in set theory) descriptions by the intrinsically dual Hilbert space such as the separable complex Hilbert space utilized by it.

The essence of the lesson taught by quantum mechanics about the way for finiteness (discreteness) not to be incomplete to infinity (continuity) consists in the option for the former to be simply doubled by duality in order to be identified with the latter, after which the internal proof of completeness, in particular, the theorems about the absence of hidden variables in quantum mechanics (Kochen, Specker 1967; Neumann 1932) are deductible. Hilbert arithmetic is suggested in other papers (e.g., Penchev 2021 August 14) so that the foundations of Hilbert mathematics are able to be complete, but in a Pythagorean manner, after which all the physical world is included in it as a necessary condition for it to be complete:

Hilbert arithmetic consists of two dual parts, called "Hilbert arithmetic in a narrow sense" and "Hilbert arithmetic in a wide sense" so that the former can be identified with the mathematical world and the latter with the physical world accordingly. Then and following Newton's testament for the "mathematical principles of natural philosophy", by which Einstein's special and general relativity can be "ontomathematically" reinterpreted as above in particular, gravitation can be understood equally well to be a physical force or interaction (as usual) or to be an ontomathematical force or interaction able to create all the physical world as dual to the mathematical world in a narrow sense (i.e., not ontomathematically, not to the mathematical world in a wide sense). This is the project of the present paper: to demonstrate gravitation as both ontomathematical and mathematical force or interaction in way consistent with its standard physical understanding.

VIII. FROM EINSTEIN'S GRAVITY TO QUANTUM GRAVITY?

The conjecture consistent with the above ontomathematical and mathematical reinterpretations of gravitation identifies quantum gravity with entanglement by mediation of quantum information as a Fourier counterpart of pseudo-Riemannian space, or speaking loosely, after substituting "time" with "frequency" or respectively continuity by two dual copies of discreteness. However, that realization of quantum gravitation is essentially different from all the class of approaches for building a theory of gravitation similar to any of the three interactions framed in the Standard model: for example, each of the latter three ones admits "secondary quantization" unlike gravitation. In other words, gravitation is "simultaneously" all the three interactions of the Standard model, each of which marked by its "proper secondary quantization" quite other than either of both rest ones therefore excluding any unambiguous secondary quantization to be relevant to quantum gravitation.

Entanglement as quantum gravitation is not still one interaction along with those three meant by the Standard model, but a dual counterpart of the Standard model as a whole and thus, of its three interactions "together". Indeed, both entanglement and gravitation in Einstein's general relativity are dual to the Standard model, but in different ways. The Standard model and Einstein's theory of gravitation are local, but the former is discrete and quantum, and the latter is continuous

obeying smooth differential equations just as those of classical mechanics. On the contrary, the Standard model and entanglement are quantum, but the former is local and the latter is nonlocal.

Both being local, the Standard model and Einstein's theory of gravitation can be called "quasi-classical" sharing the postulate of empirical and experimental locality featuring thoroughly classical science rather than only classical physics being furthermore solidly established also philosophically by Cartesianism and the organization of cognition in Modernity. Though the Standard model originates from Pauli's particle paradigm thus confessing energy conservation and unitarity (by means of which "classical" quantum mechanics can be exhaustively defined), general relativity admits violations of energy conservation and Lorentz invariance (as a counterpart of unitarity in quantum mechanics) but only in non-infinitesimal (this means finite) neighborhoods about each point of pseudo-Riemannian space.

So, both theories can be absolutely consistent since their subjects are different: the Standard model refers to infinitesimal areas of pseudo-Riemannian space, which can be granted to be "flat" thus excluding Einstein's gravitation originating from any nonzero curvature relevant only to non-infinitesimal vicinities, in which violations of energy conservation and Lorentz invariance may take place. Thus, the afore-sketched idea for the Standard model and general relativity to be relevant to each other also excludes quantum gravitation as a "mistake in definition" since "quantum" in quantum gravitation refers to infinitesimal neighborhoods, but gravitation in the same word combination means non-infinitesimal ones.

Now, one can consider, the other pair of dual theories, namely classical quantum mechanics and its nonclassical counterpart of quantum information tending to describe entanglement as a conservative generalization of the former not obeying energy conservation, unitarity, or Pauli's particle paradigm at all. If the former is local, the latter is nonlocal: thus, the former follows the postulate of not exceeding the speed of light in a vacuum, therefore able to be consistent with special and general relativity as above, but the latter does not it (a corollary sardonically emphasized yet by Einstein coining his pejorative epithet of "*spooky* actions at a distance" hinting at the conjecture is inconsistent with classical physics as an empirical and experimental science). As this is described in detail above, the theory of quantum mechanics is forced to involve non-Hermitian (thus nonunitary) operators in the separable complex Hilbert space and to generalize energy conservation postulated by classical quantum mechanics to quantum information conservation implying the option of "creation ex nihilo" as omnipresent and omnitemporal in particular.

So, two pairs of theories about dual subjects in each pair are available. Those are the infinitesimal and "flat" classical quantum mechanics versus the finite and "curve "general relativity", on the one hand, able not to disturb Einstein's spirit for are both are local and thus "unspooky" (not spooky). On the other hand, the pair of classical quantum mechanics and the theory of quantum information opposes them as local and nonlocal. One notices immediately that both pairs share the same member of classical quantum mechanics featured to be infinitesimal in the former pair, but local in the latter.

The equivalence of Einstein's gravitation and entanglement follows instantly from the identification of locality and infinitesimality embodied in classical quantum mechanics implying also and correspondingly the identification of the nonlocal with the non-infinitesimal: respectively, the finite as the non-infinitesimal after the differential and integral calculus included by Newton himself in his project of the mathematical principle of natural philosophy and then reinterpreted thoroughly physically by Einstein forced to do that by the Cartesian "fashion" and "good manners" of his age (in fact, e.g., not less conventional, inconvenient and funny than Newton's or Descartes's wigs).

In other words, if one dare restore the original doctrine of Newton without the "clothes" imposed by the Cartesian "politeness and etiquette", in fact absolutely unnecessary and awkward, as to contemporary physics for which the problem of quantum gravitation is one of the greatest scientific puzzles, the identification of entanglement and quantum gravity is a direct corollary from the ostensibly newly introduced ontomathematical viewpoint because it was Newton's good old proper design only well-forgotten and stigmatized by Cartesianism for which the mental of mathematics and the bodily of physics are gapped by an abyss, however not less redundant and ridiculous than the wigs' fashion of that epoch.

Bracketing, one might also discern that the way for interpreting the ontological compatibility of quantum and classical mechanics by scientific common sense relates their subjects (the areas of applicability) as those of the "empirical infinitesimality" (commensurable with the Planck constant) and the "finite" (commensurable with the size of the macroscopic apparatus and our everyday empirical experience). Thus, the consistency of classical quantum mechanics and general relativity as referring to infinitesimal or finite neighborhoods about any point in pseudo-Riemannian space accordingly corresponds in turn to the aforementioned ontological compatibility.

IX. QUANTUM GRAVITY: A BRIDGE FROM PHYSICAL GRAVITY TO MATHEMATICAL AND LOGICAL GRAVITY?

The last section demonstrates that the identification of entanglement and quantum gravity after granting general relativity, and classical quantum mechanics is a direct corollary from Newton's plan for "mathematical principles of natural philosophy", by means of which Einstein's special and general relativity can be easily reinterpreted only abandoning the Cartesian imperative for gravitation to be a physical force or interaction, but rather simultaneously also both mathematical and ontomathematical. The converse statement is intended to be investigated in the present section, namely: whether if one postulates the identification of entanglement and quantum gravitation as the counterpart of Einstein's macroscopic theory of gravitation, this implies Newton's ontomathematical project; in other words, whether both statements are equivalent to each other after the consideration in the previous section.

This means that entanglement and gravitation according to general relativity are adopted to be Fourier counterparts in advance, which makes physical sense only about physical quantities commensurable with the Planck constant as the phrase of "quantum gravitation" can be deciphered in the present context. Consequently, gravitation as both mathematical and ontomathematical force

or interaction relevant to our physical universe relates to the Planck scales, where the universe arises omnipresently and omnitemporally "ex nihilo" by virtue of mathematical laws and necessity. The mathematical structure and physical phenomena are closely entangled (where "entangled" may be understood in both literal and figurative senses) on the Plank scale, but can be opposed and divided by the Cartesian abyss at the scale of our everyday empirical and experimental experience from where and to which classical mechanics and Einstein's relativity originate and refer. The hypotheses of the Big Bang, Pauli's particle paradigm culminated in the Standard model, energy conservation and unitarity of classical quantum mechanics postulate wrongly that the Cartesian abyss at issue is relevant also to the Planck scale. However, that conjecture came from Descartes's age not being initially established yet in Newton's England, which allowed for him to create an alternative ontomathematical philosophy nowadays restorable secondarily as very well fitting to the quantum world.

Particularly, the strict energetic hierarchy unambiguously subordinating all entities claiming to exist is also invalid as to the Planck scale. The mental entities not possessing any nonzero energy occupy the basis of the hierarchical energetic pyramid similar to "Roman slaves" or "medieval serfs" . However, the postulate of the immateriality of all mental entities such as Plato's ideas is not true at the Planck scale. On the contrary, the prevailing in the universe "dark mass" and "dark energy" can be explained as the equivalent mass and energy of the immaterial quantum information physically acting nonlocally, but resulting locally in Einstein's gravitation after rejecting "Mach's principle" in general relativity. The basis of the visible universe is the "boundless ocean" of quantum information, among which the former is only a "small island" inhabited by humankind including in a tiny area of that "island" very far from its "shores", which is the reason for humankind even not to suspect the existence of the ocean of dark matter and dark energy until the end of the 20^{th} century. The energetic hierarchy is relevant only to the insignificant area of the "island". On the contrary, the stars and the electrons, and the ideas of Plato are equal among the primary ocean of quantum information, which they exchange nonlocally "for free" therefore generating the unity of the universe as a whole.

So, the converse statement at issue, namely that the identification of Einstein's gravity at a quantum level and entanglement in turn implies the unified physical and mathematical design of the world in Newton's manner and contradicting the usual Cartesian picture, is also true. This means that both physical and mathematical (thus non-Cartesian) basis of the world follows from the fact that the entanglement and gravitation are Fourier counterparts at a quantum level. Then, one can trace back the mathematical premises then interpreted physically for that great unification of mathematics, physics, and philosophy at a quantum level, which rather than the mythical Big Bang is the true beginning of the universe appearing from nothing by virtue of mathematical laws and necessity rather than by some alleged "Creator's will".

Gravitation after general relativity is the continuous (in fact, even smooth) Fourier temporal counterpart of classical mechanics and Einstein's relativity, to which entanglement as the discrete (quantum) "frequent twin" of quantum mechanics and information corresponds. Then, the later admits the same mathematical structure of the separable complex (respectively, qubit) Hilbert

space to be interpreted in two absolutely independent ways, which furthermore can be revealed yet among the interpretations of quantum mechanics after Born's probabilistic one: namely, in a proper quantum manner forced by the Planck constant, on the one hand, and as characteristic functions of probabilistic (density or not) distributions for any quantum quantity to be measured and crucially distinguishing it from its analogue in classical mechanics, on the other hand.

In fact, the former is the proper physical interpretation justified by the appearance and historical development of quantum mechanics as a physical theory since the beginning of the 20th century. On the contrary, the latter is the mathematical interpretation introducing and relying on an inherent mathematical theory such as probability theory. In brackets, on can notice that probability theory is implicitly non-Cartesian because the concept of probability allows for two interpretations indistinguishable mathematically: both subjective (or "mental" in Cartesian slang) and objective (or "bodily") probabilities, from where they penetrate in quantum mechanics still more confusing and entangling its interpretations since it turns out to be simultaneously an "objective" theory referring to a certain physical reality by itself and the "subjective" cognition of the experimenters and researchers about it (for example, visualizable by the "paradox of Wigner's friend"[37]).

One can discover those two inseparable counterparts originating from the always possible mathematical and physical interpretations of any theory involving Fourier transforms in the beginning of quantum mechanics to be arising from the unification of Heisenberg's matrix mechanics and Schrödinger's undulatory mechanics. Indeed, the former adhered to the literal quantum and discrete interpretation, after which any quantum states are vectors so that their changes are represented by matrices. On the contrary, the latter introduced an exotic and new concept such as "wave function" in order to describe all quantum states, and Born demonstrated soon that the wave functions of any quantum states are characteristic functions of the corresponding probabilistic (density or not) distributions of the same quantum states.

For heralding their unification, the separable complex Hilbert space were utilized since it is a special kind of Hilbert space. which shares its inherent duality to refer always both to vectors (for Heisenberg's version) and functions (for Schrödinger's version). The main idea of Hilbert arithmetic (introduced in other papers: *Penchev 2021 August 14*; etc.) borrows the same duality in order to identify it as the Gödel dichotomy relevant to the foundations of mathematics. Speaking loosely, Hilbert arithmetic extends the completeness of quantum mechanics, proved by the theorems about the absence of hidden variables in quantum mechanics (Kochen, Specker 1967; Neumann 1932) and based to the aforementioned unifying introduction of the separable complex Hilbert space, to the problem about the completeness of mathematics and its foundations (particularly and properly, by arithmetic, set theory, and propositional logic).

Thus, one can speak of Hilbert arithmetic in a wide sense sharing the same structure as the qubit Hilbert space identifiable with the separable complex Hilbert space and thus implying quantum mechanics and all the physical world as a branch of mathematics in the final analysis, on the one hand. On the other hand, Hilbert arithmetic in a narrow sense means the structure of two

[37] Wigner 1961.

dual anti-isometric Peano arithmetics as originating from the qubit Hilbert space after all qubits of the latter are considered as "empty", i.e., as the class of equivalence of all values of each enumerated qubit. By introducing that structure, physics is to be "mathematics in a wide sense" furthermore dual to "mathematics in a narrow sense". As to our everyday macroscopic experience or classical science and physics, those two versions of "mathematics" are practically gapped and can be reasonably opposed as situated on the two "shores" of the Cartesian "abyss".

However, they are merged and entangled on a quantum level commensurable with the Planck constant. The Standard model and classical quantum mechanics at all, obeying energy conservation, unitarity and Pauli's particle paradigm imply the myth of the Bing Bang in order to conserve themselves also at the quantum level at issue by summoning all anomalies, deviations and violations into the singularity of the Big Bang, in fact, inaccessible for any scientific research, but at the cost of which, all enumerated theories are proclaimed to be relevant for each moment of time strictly after it.

On the contrary, gravitation as advocated here to be the fundamental ontomathematical force or interaction endeavors to describe all those violations as obeying a much more fundamental natural law of quantum-information conservation therefore allowing for "creation from noting" to be omnipresent and omnitemporal though stigmatized by classical physics to absolutely anti-scientific and obscurantist. In fact, the myth of the Big Bang should be qualified as anti-scientific and obscurantist. Merging and entangling at a quantum level follow from quantum mechanics by itself as long it is released from the dogmas for the afore-enumerated theories to be granted for being absolutely universal: they should be restricted only to the light phase of the universe, i.e. to an insignificant part of the entire universe even not suspecting the existence of its dark phase.

Quantum information as a conservative generalization of classical quantum mechanics studying entanglement and identifying it with Einstein's gravitation[38] at a quantum level can describe all those anomalies (declaratively, but only formally prohibited by the latter) just by introducing gravitation to be the fundamental ontomathematical force or interaction able to create all the being "ex nihilo" thus absolutely not needing the "hypothesis of God". Then, quantum information can be back related to the origin of quantum mechanics by unifying matrix and undulatory mechanics visualizable, e.g., by Dirac's δ-function relatable to probability density distributions directly rather than by the mediations of their characteristic functions, i.e., wave functions of quantum states in turn being elements ("points") of the separable complex Hilbert space.

Dirac δ-function can be considered as a limit, in which "ends" a row of more and more "narrow" probability density distributions conserving their shared property for their integral square to be a unit. Thus the "height" of δ-function is infinitesimally great, but its "width" is reciprocally infinitesimally small. So, one may accept the approach of Dirac as equivalent, but alternative (being different enough) to that relied on the separable complex Hilbert space because of the

[38] A few papers consider the inherent link of information (entropy) and gravitation: Pastorello 2019; Plastino, Rocca 2018; Obregón 2015; Galperin 2011; 2011a; Verlinde 2011; Cocke, Frieden 1997; etc.

unification of Heisenberg's matrix (or vector) mechanics and Schrödinger undulatory (or wave function) mechanics and generalized as quantum information nowadays.

The basic distinction between the former (which can be briefly notated as "Dirac mechanics"[39]) and the latter consist in the following. Dirac mechanics refers to probability density distributions directly rather than to their characteristic functions as quantum mechanics does. The vector and function interpretations of the separable complex Hilbert space are opposed so that the transition between them is discrete just as that between a proposition and its negation in propositional logic. On the contrary, the more and more narrowing probability density distributions constitute a continuum smoothly transiting into δ-functions in turn corresponding to the discrete vector reading suggested by Heisenberg. So, if quantum mechanics standardly based on the separable complex Hilbert space might be conventionally seen as "contravariant" (i.e., by characteristic functions), its "covariant" counterpart would be Dirac mechanics, and entanglement respectively quantum information can be alternatively, but equivalently described by the non-zero mismatch or discrepancy (disparity) of any probability density distribution and its characteristic function also interpretable to be a quantum state "by itself" and the same after interacting nonlocally with some other quantum states, but representable locally as Einstein's gravitation.

As a conclusion of the present section, one can draw the inference that quantum mechanics (for example by virtue of the theorems about the absence of hidden variables in quantum mechanics) bridges the two shores of the Cartesian "abyss", but it has been violently restricted for about a century in the narrow framework of Pauli's particle paradigm, energy conservation, unitarity, Hermitian operators, the Standard model, etc. in order the Cartesian worldview to be conserved into quantum mechanics as well, though it by itself have gone out far beyond that framework, where, particularly, the problem of quantum gravitation has been absolutely unresolvable regardless of all huge efforts, being similar to a "round square". Indeed, the "quantum" of quantum gravitation needs that framework, but "gravitation" in the same word combinations means abandoning that framework being situated out of it.

So, if quantum gravitation is identified as entanglement, which is postulated as a premise in the present section, this implies for the afore-described framework to be broken and even absolutely destroyed. Thus, the original bridge between the two Cartesian shores being inherent

[39] In fact, Dirac suggested a very wide conception alternative to classical quantum mechanics, also called "Dirac formalism", "Dirac formulation", or "Dirac interpretation". Many papers (Gottfried 2011; Kim 2010; Gadella, Gómez 2007; 2003; 2002; Bokulich 2004; Gieres 2000; Helrich 2000; Wan, Powis 1994; Vaz, Waldyr 1993; Huang 1985; Droz-Vincent 1984; Petroni, Vigier 1983; Mishnev 1982; Antoine 1969; Dirac 1963-1964; 1954; 1945; 1942; 1939; 1937; 1925) discuss it. The problem whether it is absolutely equivalent to quantum mechanics, to its true part, or there exists some experimentally testable discrepancy between them is not investigated enough. The present paper utilizes it to explore the relation of locality and nonlocality in a still one way, which is different from both proper mathematical viewpoint of infinitesimality and proper physical approach of relativity (special and general) needing for physics to be only local and identifying locality with empirical and experimental experience after Einstein's interpretation of Mach's doctrine (though rejected by the latter), and what is especially important: Dirac mechanics is able to link the foundations of infinitesimal calculus, special and general relativity, and quantum mechanics just as the conception of "ontomathematics", advocated here, needs.

for quantum mechanics is restored due to quantum gravitation. So, and in particular, the 2022 Nobel prize in physics for entanglement and quantum information is crucial. It establishes entanglement and quantum information as absolutely legitime in science regardless of breaking and destroying the framework at issue and thus the entire organization of cognition and worldview of Modernity. Said loosely and rather metaphorically: "the 2022 Nobel prize in physics cancels or ends Modernity". It also might be seen as the ultimate final judgment in an "age-old lawsuit between Einstein and Bohr". Though the sentence formally is in favor of Bohr, it simultaneously pioneers the pathway to quantum gravity grounded on general relativity and even much further, to ontomathematical gravitation therefore restoring the original project of Newton for the mathematical principle of natural philosophy unencumbered by the Cartesian prejudice of Modernity.

X MATHEMATICAL GRAVITY ALONE BY ITSELF?

Gravitation is standardly interpreted to be a physical force and interaction along with the electromagnetic one in classical physics or those established by the Standard model in quantum physics. However, the present paper advocates a quite different, non-Cartesian viewpoint for it to be "ontomathematical", and thus both physical and mathematical. So, a corollary from the newly introduced ontomathematical status of gravitation is to be considered purely mathematically and then, even purely logically linking the usual philosophical concept of ontology with "ontomathematics" just involved. This section considers in more detail that option.

Following the pattern of general relativity after its proper geometric formulation, gravitation can be postulated by a purely mathematical definition in any "curved" vector space or respectively in any two or more subspaces of a "flat" vector space if they can be rotated to each other. Obviously, the former definition follows the literal model of pseudo-Riemannian space utilized by Einstein, but the latter means entanglement since the separable complex Hilbert space is also a vector space. Then, the unity of both sub-definitions corresponds to the conception of quantum gravitation as entanglement, and consequently, the consideration of them as Fourier counterparts, in which Einstein's gravitation is the "temporal twin" also "smooth" just as all manifolds in classical physics, but entanglement is the "frequent twin" thus discrete just as quantum physics needs.

Thus, the above Fourier duality can be generalized as relevant to any "curved" vector space (in which gravitation can be defined purely mathematically), after which it acquires a discrete counterpart, to class of which entanglement belongs in particular. Any vector space can be always doubled by an identical dual twin just as Hilbert space and the separable complex Hilbert space of quantum mechanics in particular, on the one hand. Furthermore, those identical twins possess a pair of Fourier counterparts called standardly covariant and contravariant vector space necessarily also identical in the case of a "flat" vector space, in which, consequently, gravitation is granted to be either zero or not definable. On the other hand, the same vector space can be divided into two subspaces possibly "rotated" to each other. Gravitation can be defined purely mathematically to any nonzero mismatch between the pair of contravariant and covariant vector spaces, or after any nonzero rotation of two or more vector subspaces to each other, after which the equivalence of the

ways for mathematical gravitation can be proved just as in the case of Einstein's gravitation and entanglement discussed in detail above.

However, the mathematical definition of gravitation shares the general property of all mathematical definitions in comparison with physical ones, namely, it relates to a class of equivalence rather to a certain element of that class. As to gravitation also in Newton's and Einstein's traditions and both theories of them[40], the class of equivalence at issue can be determined to all possible values of the gravitational constant, among which the special case of our universe is featured by the certain value established experimentally. Then, one might question about the mathematical sense of the gravitational constant after gravitation is able to be defined purely mathematically.

Anyway, the gravitational constant, though Newton included it directly in his equation of universal gravitation, needs the mediation of the fundamental constant of the speed of light in a vacuum to be realized purely mathematically and thus rather Einstein's approach to be returned into Newton's original ontomathematical design. So, the first stage for reinterpreting the constant of the speed of light in a vacuum consists in its realization as a certain value (valid in our universe, but now understood ontomathematically rather than physically as usual) belonging to the class of all ratios of an infinitesimal measure (unit) to a finite measure (unit). Common sense including the scientific one grants the question about that kind of ratios as inadmissible or inherently uncertain being always an infinite quantity.

However, that prejudice needs Lagrangian (language) since it is the "native" language of differential and integral calculus and then, of all mathematics, but only seemingly and a first glance. On the contrary, its closely linked counterpart of Hamiltonian (language) originates from physics and it is gapped on the other "shore" of the Cartesian abyss and any translation between the two languages is allowed only for physics. That prejudice is broken in ontomathematics (as in the present paper, foe example). So, both languages can already be utilized equally well for differential and integral calculus for defining the class of ratios between finite and infinitesimal quantities. So, they are supplied by different physical dimensions, such as distance and velocity where the latter is the first derivative of the former, since Hamiltonian language appeared historically first in physics rather than in mathematics, but this is an occasional and unessential fact (a QWERTY effect). So, their ratio also possesses a physical dimension, that of time, and that property can be postulated for the definition of time purely mathematically as below.

In other words, the class of ratios featured unambiguously by the speed of light in a vacuum as to our universe means relativistic time of special relativity being grounded on the exchange of light signals as in Einstein's famous thought experiments, for example, about the physical sense of simultaneity in special relativity. The proper mathematical meaning of the constant of the speed of light in a vacuum can be then elucidated by the ratio of finite distance (usually notated as "x") and infinitesimal distance (usually notates as "ct" where "c" is the speed of light in a vacuum, and

[40] For example, cf. Enosh, Kovetz 1973.

"t" is time), which is to be rather interpreted as an infinitesimally great distance ant its reciprocal value, "$\frac{1}{ct}$" would be an infinitesimally small distance.

One might further question how the above description expressed in Hamiltonian (language) being inherent for physics originally is to be translated into mathematics where the concept of physical dimension is not available. It is to be substituted by "mathematical dimension", moreover it is natural and definitive in relation to any vector space, within the class of which purely mathematical gravitation is intended to be introduced. So, if one considers any two axes of any vector space, they are conventionally granted to possesses equal "scalar units" being distinguishable only as unit vectors, orthogonal or arbitrarily rotated to each other in general. However, this is only a convention which can be easily generalized so that any two axes of any vector space to be arbitrarily scaled for example by the parameter "c", i.e., by the value of the speed of light in vacuum, but substituting its physical dimension of velocity by the mathematical dimension of any other axis scaled with the dimensionless parameter equal to "c". One can conclude that the description in Hamiltonian (language) in mathematics in a narrow sense (i.e., not in ontomathematics) is both correct and absolutely equivalent to that in physics.

Granting the structure of vector space, one can define the concept of information both classical measured by bits and quantum measured by qubits in relation to that structure as follows. The choice of either of any two axes of a vector space corresponds to a bit information. At that, one should distinguish here two oppositions as usual: (1) before the choice versus after the choice; (2) the proper choice of either of any two axes. Then, one can notice that the class (but not the set) of all axes of any Hilbert space (thus including the separable complex Hilbert space of quantum mechanics or the qubit Hilbert space of quantum information) obeys the axioms of Peano arithmetics, on the one hand, and each of them doubled by its dual counterpart constitutes a bit of information, so that the class of all axes including their dual counterparts can be considered as an enumerated sequence of bits of classical information, on the other hand.

On the pair of any two axes of any vector space one can define a qubit as the normed superposition of them where the two norming coefficients belong to field on which the vector space itself is defined so that if the case is the particular one of the separable complex Hilbert spaces, the above definition coincides with that in quantum mechanics and information. In other words, the suggested definition on any vector space is a generalization of the standard one. Minkowski space is the Fourier counterpart of the qubit Hilbert space. So, the definition of "qubit" on any vector space can be utilized to be defined as the Fourier counterpart of the vector space at issue by means the Fourier counterpart of the so-defined generalized qubit in that vector space.

Meaning the preceding consideration of time to be defined purely mathematically as the Hamiltonian ratio of the infinitesimal and the finite, one can reinterpret also the gravitational constant as defining a certain amount of mass (or respectively energy), thus featuring any entity claiming to be physical corresponding to a qubit. That can be visualized by the physical dimension of the gravitational constant being "$x^3.t^{-2}.m^{-1}$". So, if "t" is defined in advance as a little above to be the "infinitesimal Hamiltonian ratio", the gravitational constant determines that mass

corresponding to the volume "v^3" in Minkowski space, so that the Fourier counterpart of which in the qubit Hilbert space is an "empty qubit".

Then one notices that gravitational constant available yet in Newton's equation of universal gravitation, but interpreted as in the previous paragraph by means of quantum information, corresponds exactly to his implicitly non-Cartesian project of the mathematical principles of natural philosophy, after which there should exist a coefficient such as the gravitational constant equating mass (respectively energy after Einstein's "$E = mc^2$"), i.e., a universal physical quantity, with information or quantum information, relevant rather to the Cartesian "mind" or Platonian "ideas", i.e., a universal mathematical variable. However, that sense of the gravitational constant though natural for Newton has been gradually lost thoroughly for the more and more dominating Cartesian prejudice after which the theory of universal gravitation is physical rather than ontomathematical since the concept of ontomathematics itself were canceled.

The consideration until now includes the constant of the speed of light in a vacuum and gravitational constant, but not the Planck constant. The former two constants correspond to Einstein's special and general relativity in turn interpretable in the direction of Newton's design called here ontomathematical. However, Newton created also the corpuscular theory of light[41] rediscovered by Einstein in his theory of photoelectric effect: that was the formulation for which discovery he won the Nobel Prize in Physics for 1921 (not for special or general relativity).

The Planck constant can be directly interpreted as a an ontomathematical ratio determining the physical action corresponding to a qubit, and absolutely independently of the other two fundamental constants meant above. Speaking loosely, one can say that the former fundamental constant is "more fundamental" than the latter two ones in a sense: it refers to both local, "light" and nonlocal, "dark" part of the universe, but the gravitational constant and the speed of light in a vacuum relates to what local observers such as humankind can watch from the own inherent local viewpoint and the reference frames linkable to it.

Those two "less fundamental" constant are interpreted to be rather local and thus relevant to about 4-5% of the total mass and energy of the universe though physics until recently identified those "light" 4-5% with the entire universe. They mean for any quantum states to be in an absolutely decoherent state: the local to be completely divided from the nonlocal, which is the proper physical sense of the speed of light in a vacuum and after which a certain amount of energy (respectively, mass) corresponds to a qubit due to the gravitational constant. In other words, the (incorrect) postulate for them to be equally fundamental as the Plank constant is also equivalent with classical quantum mechanics, energy conservation, unitarity, or Pauli's particle paradigm, and thus, with the Standard model in drawing things to a close.

Otherwise said, time is not divided from energy (or mass) in any coherent quantum state or in the dark phase, so that both are able to be directly transformed into each other therefore violating energy conservation, but observing quantum-information conservation, which both light and dark phases of the universe obey and originating in a sense from the Planck constant being the most fundamental one. As this is very well known, the three constants together are able to generate all

[41] For example, Darrigol 2010 or Ziggelar (1993).

physical dimensions relevant to classical mechanics, which can be demonstrated by the "Planck length, time, and mass". Consequently, if the gravitational constant and the speed of light in a vacuum are not relevant to the dark phase of the universe, only physical action and information or quantum information are inherent there. However, one might anyway involve an auxiliary representation following the usual physical picture borrowed from classical mechanics and including all the usual physical quantities definable, but suspending energy conservation and substituting it with quantum information conservation.

If one complements that approach with the requirement for any violation of energy conservation to be equivalently represented by the addition of a certain "elementary particle" participating in the interaction at issue, one would utilize Pauli's particle paradigm (or Feynman's diagrams) in particular. Then, the natural question is: why gravitation is impossible to be included consistently in that physical worldview dominating until now after any violation of energy conservation being inherently nonlocal can be anyway equivalently represented locally by complementing relevant elementary particles in order to keep energy conservation.

Though this is the intention of classical quantum mechanics attempting for about a century to create a relevant theory of quantum gravitation in those frameworks, the following consideration contradicts it. Gravitation only doubles globally the same physical picture of the world, being local in definition and suggested in detail by the Standard model. So, gravitation cannot be included locally within itself since any idea for that is self-contradictory. The true answer might be that quantum gravitation is not able to be inserted strictly within any conservative generalization of the Standard model since it is a dual representation of the Standard model as a whole. The entanglement theory of quantum gravitation highlights that duality especially discernably.

Classical quantum mechanics, as well as classical mechanics or special and general relativity are thoroughly within the Cartesian organization of cognition, after which mathematics and physics are gapped, and any ontomathematical approach to gravitation (which might be at least suggested after Newton) is rejected in definition as nonsense, out of science in principle. However, the present paper advocates the opposite viewpoint possibly restoring Newton's original design. Then, and in particular, the ontomathematical approach of gravitation to be considered purely mathematically thus definable onto any vector space, and Einstein's geometrical theory of gravitation can illustrate very elaborately it.

The next section will make step further: to the option for gravitation to be discussed only logically on the same ontomathematical basis. That approach can be also interpreted as returning into the logical origin of non-Euclidean geometry after Lobachevsky investigating the theory and logical sequences after substituting the Fifth Postulate with its negation (though initially inspired tosuggest a proof of the Fifth Postulate by *reductio ad absurdum*). On top of that, Hegel's dialectics and dialectical logic are codirectional to the ontomathematical idea of purely logical gravitation[42].

[42] Hegel's dialectics though interpreted by himself as the natural ontology of the world and thus as natural philosophy (for which it has been often criticized for being scholastic, metaphysical and anti-scientific) can be anyway rehabilitated partly in the present context as a continuation of Newton's implicit ontomathematics, however, realized as universal ontology in the talweg of the philosophical tradition; some papers which may be cited are: Borzeszkowski 1993; Buchdahl 1993; Burbidge 1993; Buttner 1993; Drees

XI. LOGICAL GRAVITY ALONE BY ITSELF?

The smooth transition to "logical gravitation" once mathematical gravitation is granted in advance, for example, after the consideration in the last section can be formally represented as follows. Propositional logic being the first-order logic shared by all possible first-order logics, i.e., mathematical theories, is featured by a finite tuple of axioms transferred without any change as to all first-order logics, each of which is featured by an additional tuple of axioms. The distinction between zero-order axioms and first-order axioms is more or less conventional so that any axiom can be accepted also conventionally to belong to the former tuple containing only zero-order axioms therefore transforming it into an additional zero-order axioms.

The newly introduced tuple of axioms, containing the additional axiom, can be qualified as a nonclassical logic often interpreted to be a "logic of something" where "something" serves to notate some first logic relating that "something" at issue, which is the purely mathematical theory of gravitation in the particular case in the present paper. Then, if one conveys suitable axioms able to describe gravitation mathematically to the newly introduced zero-order "logic of gravitation", purely logical gravitation would be defined. In fact, that transition, though historically realized in the opposite direction, is that one: from Lobachevsky's to Riemann's approach to non-Euclidean geometry. So, the sense of logical gravitation being inferred from mathematical gravitation consists in the substitution of the space curvature parameter introduced by Riemann to the logical opposition of the Fifth Postulate to its negation as Lobachevski did.

So, Lobachevsky's approach can be utilized to Minkowski space, which being "flat", though "concave", can be discussed analogically to Euclidean space also "flat", but "convex", In fact, Minkowski space can be interpreted as Euclidean space, in which a spherical light wave propagates obeying the postulate of the constant speed of light in a vacuum therefore dividing it into two alternative domains both limited by the light cone, correspondingly "below" and "above" and usually notated as the real and imaginary areas of Minkowski space. Consequently, the strict and unambiguous analogue of the Fifth Postulate to Minkowski space can be suggested only substituting the concept of "straight line" with that of "spherical manifold" and visualizable as usual balls in Euclidean space. Then, the relevant analogue of the Fifth Postulate also being equivalent to its "flatness" (regardless of "concaveness") would state, that there exists only a single spherical symmetry able to transform a certain spherical manifold belonging to the light cone into another determined by any point of the light cone belonging to the transformed spherical manifold. The analogue of the Fifth Postulate relevant to Minkowski space can be visualized by concentric balls in Euclidean space and the statement that if a certain ball and a certain point not belonging to its surface are given in advance, there exists only a single ball concentric to the given one such that the given point belongs to it.

1993; Engelhardt 1993; Falkenburg 1993; Fleischhacker 1993; Garrison 1993; Gjertsen 1993; Gower 1993; Grattan-Guinness 1993; Ihmig 1993; 1993a; Illetterati 1993; Kluit 1993; Melica 1993; Miller 1993; Morretto 1993; Petry 1993; Pozzo 1993; Priest 1993; Snelders 1993; Toth 1993a; Wahsnerin 1993; Wandschneider 1993; Wehrle 1993; Weinstock 1993; Wolf-Gazo 1993.

One can further trace Lobachevsky's approach applied to Minkowski space by rejecting the afore-formulated analogue of the Fifth postulate similarly allowing for either no concentric symmetry to exist (accordingly, following rather Riemann), on the one hand, or more than one concentric symmetry to exist (thus following Lobachevsky and his hyperbolic counterpart of Euclidean geometry literally), on the other hand. Next, a parameter analogical to Riemann's space curvature can be introduced so that each "non-Minkowski" (i.e., replacing "non-Euclidean") geometry us featured unambiguously by a single real value of the parameter at issue. If one admits that parameter to be variable so that space curvature is changeable from a point of the considered geometric manifold to another, the resultative vector space may be identified with pseudo-Riemannian space of general relativity, by which gravitation can be defined purely mathematically as above.

Following the Einstein field equation, one can exhaustively define gravitation by two different pseudo-Riemannian spaces such that the one of them allows for the energy momentum tensor to be unambiguously determined in any point of it, and the other one refers similarly to the relevant spacetime tensor. Purely mathematically, the two different pseudo-Riemannian spaces though physically clearly distinguishable by their physical dimensions (spacetime for the latter, but energy-momentum for the former) can be identified with the change of a single pseudo-Riemannian space and thus as an operator acting upon it. The case of two physically absolutely different pseudo-Riemannian spaces corresponds to the light phase of the universe after which energy (respectively energy-momentum) and time (respectively spacetime) are absolutely divided form each other and just that is the case granted to be both exhaustive and in default as such by classical mechanics, special and general relativity, and classical quantum mechanics, even by all physics until recently.

On the contrary, the dark phase of the universe can be featured by the latter interpretation being necessarily complemented by the Fourier counterpart of the Einstein field equation to be relevant to the entanglement theory of quantum gravitation. The Einstein field equation itself should be supplemented by members corresponding to the mutual transformation of the spacetime and energy-momentum tensors for it to be generalized to the cases of the "non-orthogonality" of space-time and energy-momentum, in particular conditioning the option for creating from nothing, being absolutely inadmissible for classical science and physics and opposing them to religion. The quantitative ontomathematical laws, which "creation ex nihilo" obeys, is a new and a very essential and even crucial step of science onto the territory, which religion reckoned until soon to be monopolistically reserved for itself. Those members missing in the original Einstein field equation furthermore refute "Mach's principle", but their effects are locally observable just as gravitational and caused by "dark mass" and "dark energy" being inherently "dark" for their nonlocal nature. Just they correspond to the missing members in the Einstein field equation, however, quite incomprehensible, since their existence, though prevailing in the universe, did not even suspect in his time.

The same equation and the corresponding pair of pseudo-Riemannian spaces are returned to the analogue of Lobachevski's approach relevant to Minkowski and "non-Minkowski" geometries.

Gravitation is zero for the former, “flat” space, but nonzero for the latter just by virtue of general relativity or its eventual generalization by an entanglement theory. Then one might say in a Hegelian manner that the “dialectical contradiction” of the Fifth Postulate and its negation is resolved by the “synthesis” of the “thesis” of the former and the “antithesis” of the latter just as dialectical logic claims. Thus, it tends to explain gravitation as a manifestation of the universal laws of dialectical logic. A commonly accepted objection against dialectical logic consists in the blame that it is able to explain anything, but predict nothing. It served for Popper to create his conception about the demarcation line of metaphysics (explaining anything, but predicting nothing) and science able to explain and predict only certain phenomena, but definitely not all of them.

However, meaning the special case of gravitation as a manifestation of the universal Hegelian dialectical contradiction, one may consider the converse problem whether any dialectical contradiction is able to be represented quantitatively (i.e., by means of the Einstein field equation and its generalization by an entanglement theory of gravitation). The idea of purely logical gravitation discussed in the present section means its positive answer. That is, any logical contradiction might be resolved in the sense of dialectical logic introducing gravitation and all the physical world generated by it.

Indeed, if any dialectical contradiction is given in advance, one can apply the afore-described method by the pattern of Minkowski and non-Minkowski geometries resulting into pseudo-Riemannian space and then, into a certain gravitational field. The usual approach of any paraconsistent logic endeavoring to model formally dialectical logic is quite different, since it is a nonclassical logic claiming to replace propositional logic as the universal zero-order logic of mathematics, science, and rationality at all. That approach follows Hegel’s testament for dialectical logic to be granted as a new and better zero-order logic relevant equally well to “subject” and “object” to both “body” and “mind” unlike classical propositional logic not being ontological.

On the contrary, the purely mathematical interpretation of gravitation, deducible (as in the last section) from the advocated here ontomathematical approach to it, is a mathematical theory and thus a first-order logic. Then, purely logical gravitation can be interpreted as the same mathematical theory after transferring its axioms from the sub-tuple featuring just the first-logic at issue into the fundamental sub-tuple of the axioms of a newly introduced special nonclassical logic, which can be called provisionally “logic of logical gravitation”, and then in particular to be investigated its relation with Hegel’s dialectical logic, which in turn is a rather philosophical, ontological and speculative doctrine than a mathematical or scientific theory.

Meaning general relativity as a first-order logic thus reducible only to operators (tensors) acting onto pseudo-Riemannian space, the proper logical gravitation can be understood as the identification of Hegel’s “dialectical contradiction” as the duality of covariant and contravariant spaces, and their mismatch equivalent to a nonzero quantity of gravitation, resulting in their “synthesis”, which is the nonzero quantity of gravitation at issue. The last consideration elucidates that Hegel’s dialectical logic is an only qualitative theory thus unable for any quantitative predictions or for the distinction of true and false compose propositions as classical propositional

logic. The level of cognition in Hegel's age was not sufficient for the idea of dialectical logic to be embodied into a really scientific theory whether physical, mathematical, or logical. Though being an only philosophical idea, Hegel's dialectics pioneered the pathway to many "falsifiable" (in Popper's sense) scientific ideas nowadays, among which the discussed concept of "purely logical gravitation" is situated in particular. The proper contribution of dialectical logic being a special philosophical ontology consists in the gerne of ontomathematics, which it contains.

For example, whatever vector space implies the duality of relevant covariant and contravariant spaces, after which its "flatness" is equivalent to their coincidence, and accordingly, the purely mathematical gravitation is equivalent to their mismatch resulting into the "curvature" of the initial vector space. The corresponding covariant space can be interpreted as the space of the measured physical quantities (or physical theories) thus necessarily "after measurement" (in quantum mechanics's "slang"), and its covariant counterpart means the space of the units for measuring (or mathematical models, to which mathematical theories and thus first-order logics belong in turn) thus necessarily "before measurement" (again in quantum mechanics's "slang"). Then, gravitation once it has been in advance understood ontomathematically links physical reality (or the Cartesian "body", or the "object" of classical German philosophy), though as physical theories including empirical experience and more or less complicated and sophisticated experiments, and mathematical models for it (or the Cartesian "mind", or the "subject" of classical German philosophy), therefore being able in particular to describe the way for generating physical reality from the "nothing" (or from the "immaterial information" of mathematical models). Obviously, this is an idea absolutely inconsistent with classical science, which stigmatizes it as anti-scientific, mystic, or religious.

Next, the same vector space meant in the previous paragraph can be always supplied with a Fourier counterpart usually a single one, then interpretable to be discrete mathematically or quantum physically by the simple formal substitution of the variable of "time" by that of "frequency" implying furthermore the replacement of the concept of "physical change" as a temporal process by that of the simultaneous and synchronic "probability density distribution" of quantities such as those studied by quantum physics. One may use the very well elaborated pattern of the separable complex Hilbert space furthermore visualizable by its interpretation in quantum mechanics:

Then, duality can be interpreted as negation in propositional logic (rather than in any version of quantum logic) where "dialectical contradiction" would correspond to entanglement identifiable with Einstein's gravitation after the entanglement theory of gravitation. Thus, the Fourier counterpart of purely logical gravitation is verified once again and absolutely independently therefore confirming the mutual consistency of the whole ontomathematical design of gravitation traceable to Newton.

XII INSTEAD OF CONCLUSION: CREATION BY GRAVITY, BUT WITHOUT GOD?

Modernity (especially science) glorifies itself to be emancipated from religion and its occasional myths being arbitrary human fictions thoroughly results of imagination and drastically contradicting scientific experience and experiments. The Cartesian revolution freed science from

the shackles of religion by gapping the bodily empirical experience from any mental construction including those involved by religion and theology furthermore establishing the former as the ultimate arbiter in relation to the former. Following experimental natural science, humankind created and creates more and more exponentially newly and newly technologies and technics exceptionally facilitated human life only for a few centuries.

However, the rapid development of science led in the eve of the new millennium to the limit of locality being identifiable with the Cartesian organization of cognition, in which mathematics and physics are gapped in particular, after revealing entanglement, "dark matter" and "dark energy". Though the emancipation of science relying only on empirical (and thus local and "light" in default) experience and experiments reached those huge successes and continues to do it, the Cartesian "abyss" was only a human postulate, i.e., convention very useful and conditioned the progress in Modernity, which exhausts now or will exhaust its potential very soon. The expanses of the universe and knowledge turn out to be rather nonlocal and far beyond classical science and its Cartesian prejudice:

On the contrary, physics and mathematics are the same and their single unity includes furthermore philosophy, and it is called ontomathematics as a neologism following the pattern of "ontology". The subject of the present paper is restricted only to the ontomathematical interpretation of gravitation revealing its roots after Newton, Einstein, and quantum mechanics and information. Once gravitation has been understood ontomathematically, it is the omnipresent and omnitemporal creating source of the universe after abandoning "Mach's principle" in particular.

One might say that the Big Bang is a Cartesian myth summoning all the overlapping of "body' and "mind" in the beginning of the universe, after which it is already exemplarily Cartesian and particularly ready for classical science. If Christianity needs God's creation according to the Bible so that the world obeys God after it, classical science, understood widely including classical mechanics and Einstein's special and general relativity, needs the Big Bang after which the universe obeys it, but not less imaginarily than the world obeys God in the former case. The myth of the Big Bang is not much better than the biblical one, that of God's creation. Both myths fill the vast gaps of ignorance with fictions just as the human mind completes the picture of the world with more or less loose extrapolations and conjectures weakly originating from the real perception.

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