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New Method of 1-D to 2-D Transformation of Time Series

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Abstract: New method of 1-D to 2-D transformation of time series or one-dimensional signals was proposed. The method is based on a binary transformation of the time series. Resulting binary points are considered as interacting pseudo particles. The transformation is carried out using moving window. In each position of moving window, the pseudopotential is calculated and is written in the central point of moving window. Two pseudopotentials were considered: Ising spin-spin potential and Coulomb potential. Both potentials equally well describe the features of time series dynamics. Potential calculations were carried out at different values of the width of the sliding window. The resulting potential curves were grouped together to create 2-D images. This 1-D to 2_D transformation was compared with continues wavelet transform (CWT). The comparison showed that potential transformation gives more detailed information of hidden patterns then CWT.

Key words: time series, continues wavelet transformation, Ising potential, Coulomb potential, binarization, normalization, detrend, clearing time series from outliers

Introduction. One dimensional time series (TS) are the discrete sequences of different kind objects. Let us consider numerical time series (NTS). Relative location

of points inside NTS may be considered as one-dimensional pattern. Two-dimensional gray scaled images are two-dimensional patterns of bytes. From the classical point of view TS is usually considered as addition of regular signal and noise [1]. In such approach the main problem is to get rid of noise. To solve the problem, one can use smoothing algorithms [2] and filters. Various information may be obtained using the continues or discrete wavelet transform [3-5]. Continues wavelet transformation (CWT) is the transformation of TS from 1-D to 2-D representation. Thus, this transformation widens the horizons of TS analysis. Discreet wavelet transformation (DWT) separates TS into several parts: approximation and details [3-5] Our point of view is that noise patterns sometimes contain significantly useful information. Patterns are mosaic of relative location of TS points. So, to analyze patterns one needs distance calculation. In image processing the distance transformation (DT) is widely used as a useful tool for pattern analysis [6]. DT proved to be useful in many practical applications. There are many different methods and distance measures which are used in DT calculations. Euclidian DT (EDT) is one of the popular distance measure [7]. The drawback of EDT algorithms is that EDT algorithms are slow-speed leading to different problems. For example, the problem of sparse object representation in discrete geometry. This problem was considered using DT in [8]. The DT algorithm was also used in [9] for automatic pattern recognition. The DT algorithm complexity was analyzed in [10], in which several effective algorithms were developed: Linear-time Legendre transform (LLT) algorithm, the parabolic envelope (PE) algorithm and non-expansive proximal mapping (NEP) algorithm. It was shown in [10] that these algorithms have linear complexity and may be effectively used for DT processing of images. Nowadays the high-speed parallel computing and GPU computing are often used in DT calculation [11]. DT is useful in many practical applications. For example, in medical application DT is one of best means for discovering the similarity in image series [12]. It is rather important for inner organs slice-by-slice image analysis. Good results were obtained using together watershed algorithm and DT for blood cell image segmentation [13]. Watershed algorithm needs grayscale images. In [13] DT transform was used to transform binary image to gray-scale. Four distance measures were used:

EDT, city-block, chessboard and quasi-Euclidean. It was found that chessboard DT measure had best results in watershed segmentation.

In our present work we used DT for so called pseudopotential calculation. Let us assume that normalized TS was binarized, for example, on 0.5-level. Let us call points for which $f(x_i)$ > level as "white" points, otherwise "black" point. Sometimes they are called feature or background points. The "white" points in binary image may be considered as pseudo particles. These particles create pseudopotential field which value may be put into the central pixel of moving window (MW). In one pass of MW on the time series we get one potential curve corresponding to the chosen width of MW. By grouping together these curves for different values of MW-width one obtains 1-D to 2-D transformation of TS. This transformation is like wavelet transform but has different nature. While wavelet depends on one parameter, potential transform depends of whole points configuration. We considered two kinds of pseudopotentials: Ising spin-spin interaction potential [14] and Coulomb potential. Other kinds of potential may be considered, for example, and Lennard-Jones potential [15,16] or Tersof [17] or Morse [15,16] potential.

Potential transformation

For potential transform one needs the TS preliminary processing and binarization.

1. Preliminary processing.

1.1. The first step of preliminary processing is the removal of trend. It is possible to model trend using both local or global smoothing or approximation. In our study we used local spline regression. We get the detrended TS as follows $v_i = a_i - f(a_i)$, where a_i - initial TS, $f(a_i)$ - trend.

1.2. Second step is the removal of outliers. It is important to remove the outliers because they are more anomalies and are not related to the patterns. To remove outliers we use the following algorithm: (a) calculation of global average and standard deviation; (b) deleting from TS points with values greater then, two standard deviations: $k = \arg(|v_i| > 2\sigma)$, $v_k = [\,]$, where [ ] – means "delete", σ - standard deviation; (c) new calculation of global average and standard deviation; (d)

normalization of resulting TS $u_k = \frac{v_k + 3\sigma}{6\sigma}$, where σ - new standard deviation (normalization transforms TS interval $[-3\sigma, 3\sigma] \to [0,1]$); (e) additional normalization as follows: $x_k = \frac{u_k - u_{\min}}{u_{\max} - u_{\min}}$.

2. *Binarization*

Binarization is the transformation of TS into binary points

$$b_k = \begin{cases} 1, & x_k > L, \\ 0, & otherwise \end{cases} \quad (1)$$

where L – binarization level.

Let us name points, for which $b_k = 1$ "white" points and otherwise "black" points.

3. *Potential transformation*

The potential transformation is performed using so called moving window (MW). The MW moves along TS from one point to another. The potentials are calculated only for "white" binary points inside MW. In the present work we used two kinds of potential: distance dependent potentials (Coulomb potential) and distance independent Ising spin-spin potential [14].

3.1. *Ising potential transform (IPT)*

In Ising model the spin-spin interaction is considered only between nearest spins [14]. Let us assume that "white" points have spin $S_i = 1$ and "black" points have spin $S_i = -1$. The total Ising potential is equal to the summation of all two-particle interactions:

$$U_{Ising}(c) = \begin{cases} J \sum\limits_{(t,q \in MW)} S_t S_q = J \sum\limits_{(t,q \in MW)} b_t b_q, & r_{tq} \le R, \\ 0, & otherwise \end{cases} \quad (2)$$

where: *MW* – moving window; *c* – central point of moving window; *t*, *q* – "white" points inside moving window; r_{tq} - distance between points *t* and *q*; *R* – limits of Ising interaction; *J* – energy constant (in calculation *J* = 1).

In every position of MW, the total potential of spin interaction between particles is assigned to central point. The resulting TS was called Ising potential transform (IPT).

If $R = w$ (width of MW), then $U_{Ising}(c) = \frac{1}{2} n_{tq}(n_{tq} - 1)$, where n_{tq} is the number of "white" points inside MW.

3.2. *Coulomb potential transform (CPT).*

The algorithm (CPT-algorithm) uses total Coulomb potential of interactions between "white" binary points $U(p) = \sum_{i<j} V(r_{ij}) = \sum_{i<j} \frac{1}{r_{ij}}$, where: r_{ij} - distance between two binary points. We compute the total interaction between "white" points as follows:

$$U(c) = \sum_{i,j \in G} \frac{1}{|i - j|}, \tag{3}$$

where: $G = \{i \neq j \neq c;\ i, j, c \in MW\}$ MW – moving window; c – central point of moving window.

Other kinds of potential may be used, for example, Lennard-Jones potential [15,16]:

$$u(r_{ij}) = \begin{cases} 4\varepsilon \left(\left(\frac{\sigma}{r_{ij}} \right)^{12} - \left(\frac{\sigma}{r_{ij}} \right)^{6} \right), & r_{ij} < r_c \\ 0, \quad otherwise \end{cases} \tag{4}$$

where r_{ij} - distance between binary points, ε, σ - potential parameters, r_c – cut off distance (potential limits). The parameter ε defines the strength of the interaction and σ is a length scale. The interaction repels points at close range, then attracts, and is eventually cut off at some limiting distance r_c.

Assuming $\varepsilon = 1, \quad \sigma = r_c = w$ and taking d as half MW width we get the following calculating equation for total Lennard-Jones potential:

$$U(c) = 4 \sum_{r_{ij} < w} \left(\left(\frac{w}{|i - j|} \right)^{12} - \left(\frac{w}{|i - j|} \right)^{6} \right) \tag{5}$$

where w – is the width of moving window.

We assume that using another kind of particle interaction, for example, the Morse potential or Tersoff potential [15-17], one may get another hidden pattern.

For each position of MW, the potential values of different binary slices constitute potential vector. We used this vector for statistical indices calculation which we consider as TS structure signatures. We calculated the following normalized statistical indices:

Each individual potential curve corresponding to a chosen window width may be analyzed using following statistical indices:

Normalizes average:

$$\overline{p} = \frac{\frac{1}{n}\sum_{i=1}^{n} p_i}{\max_i\left(\left|p_i\right|\right)}, \tag{6}$$

where p – potential vector, n – number of TS points.

Normalized standard deviation:

$$\sigma = \frac{\frac{1}{n-1}\sum_{i=1}^{n}\left(p_i - \overline{p}\right)^2}{\max_i\left(\left(p_i - \overline{p}\right)^2\right)}, \tag{7}$$

Normalized skewness:

$$Sk = \frac{\frac{1}{n}\sum_{i=1}^{n}\left(p_i - \overline{p}\right)^3}{\max_i\left(\left|p_i - \overline{p}\right|\right)^3}, \tag{8}$$

Normalized kurtosis:

$$Ku = \frac{\frac{1}{n}\sum_{i=1}^{n}\left(p_i - \overline{p}\right)^4}{\max_i\left(\left(p_i - \overline{p}\right)^4\right)}. \tag{9}$$

Results

As example for potential transformation let us consider time series of stock prices of platinum of London Metal Exchange. The data was obtained on the website https://www.quandl.com/data/LME. The data cover the interval from 1990 to 2018.

Fig.1 show the initial TS and its spline approximation.

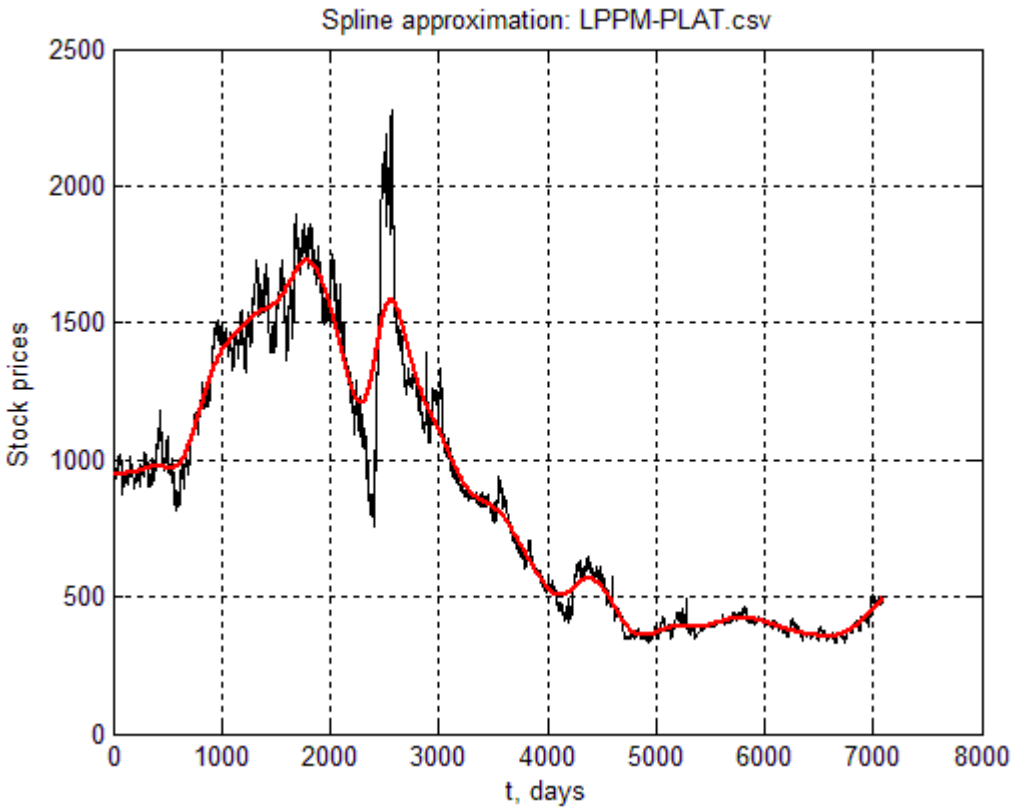


Fig.1. Initial TS – black and spline approximation - red: stock prices of platinum of London Metal Exchange from 1990 to 2018

Big outlier is located near 2500 day. Next step is trend removal. The resulting detrended TS is shown in Fig.2.

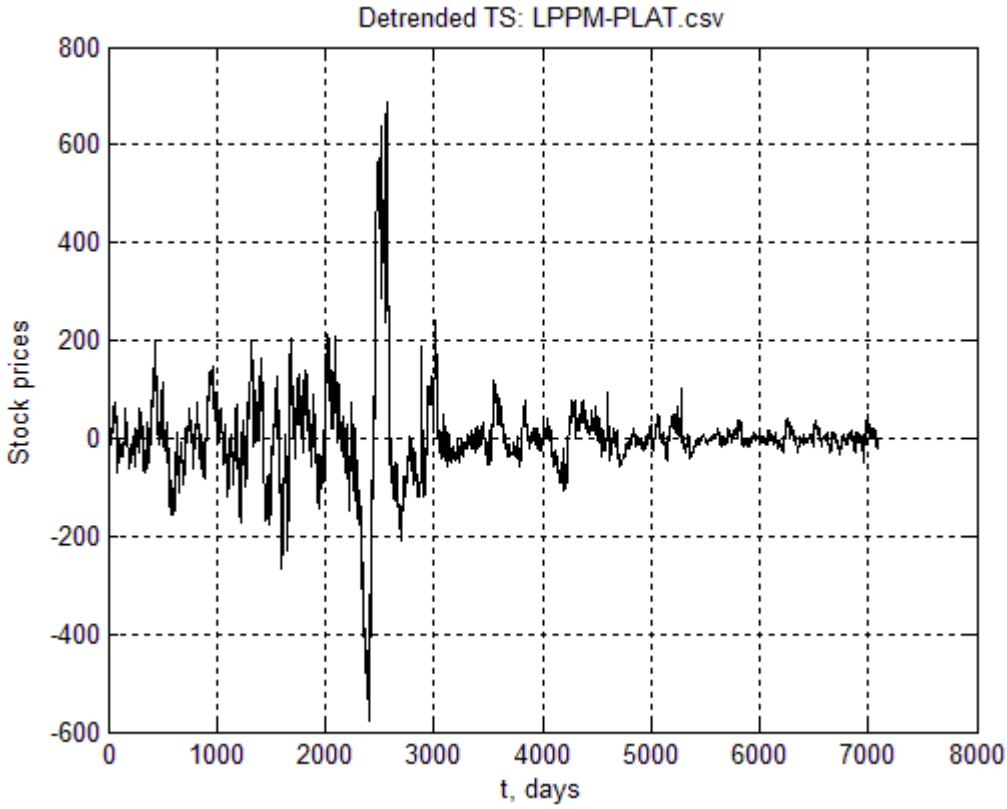
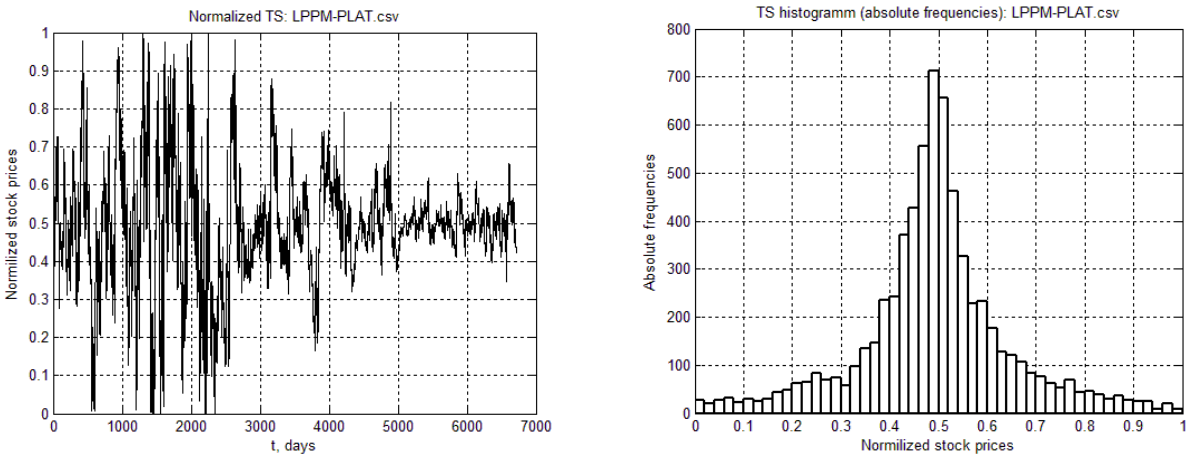


Fig2. Detrended TS.

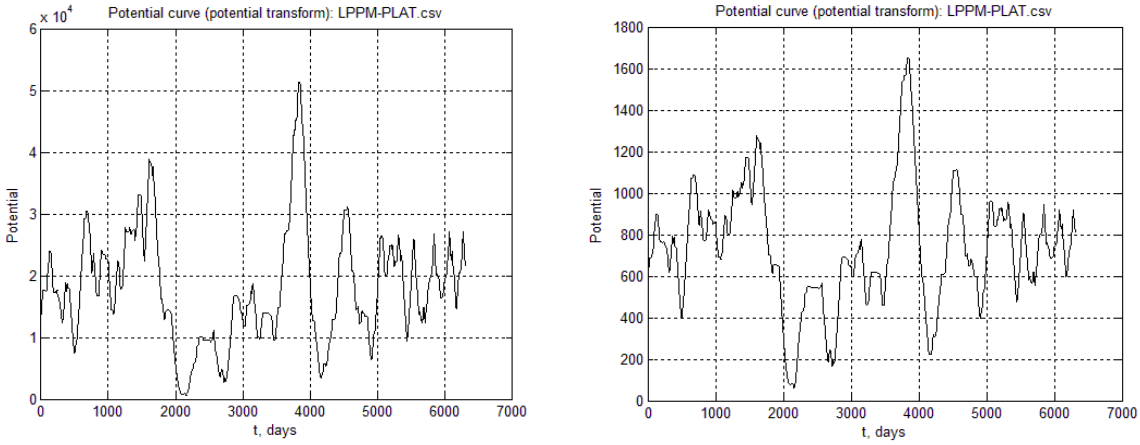
It is clear from Fig.2 that outlier problem is not solved. Clearing detrended TS and normalizing it as is described in section 1.2 we get cleared TS shown in Fig.3.



a b

Fig.3. Normalized cleared TS and its histogram

Cleared TS was used in potential calculation. One of the Ising potential curves and one of the Coulomb potential curves are shown in Fig.4.

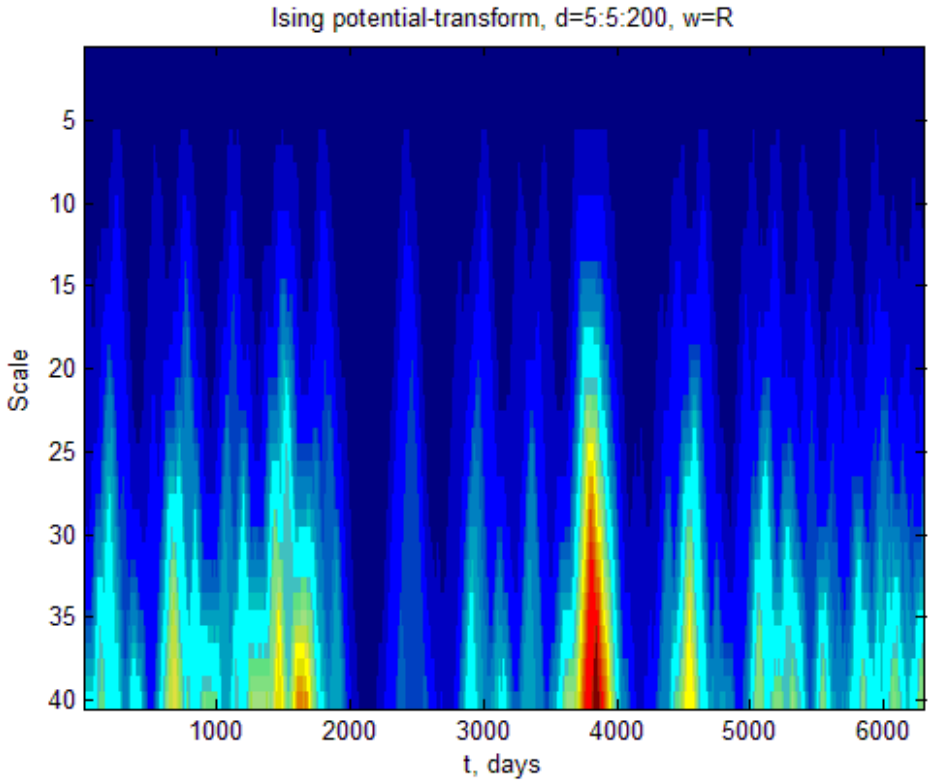


a b

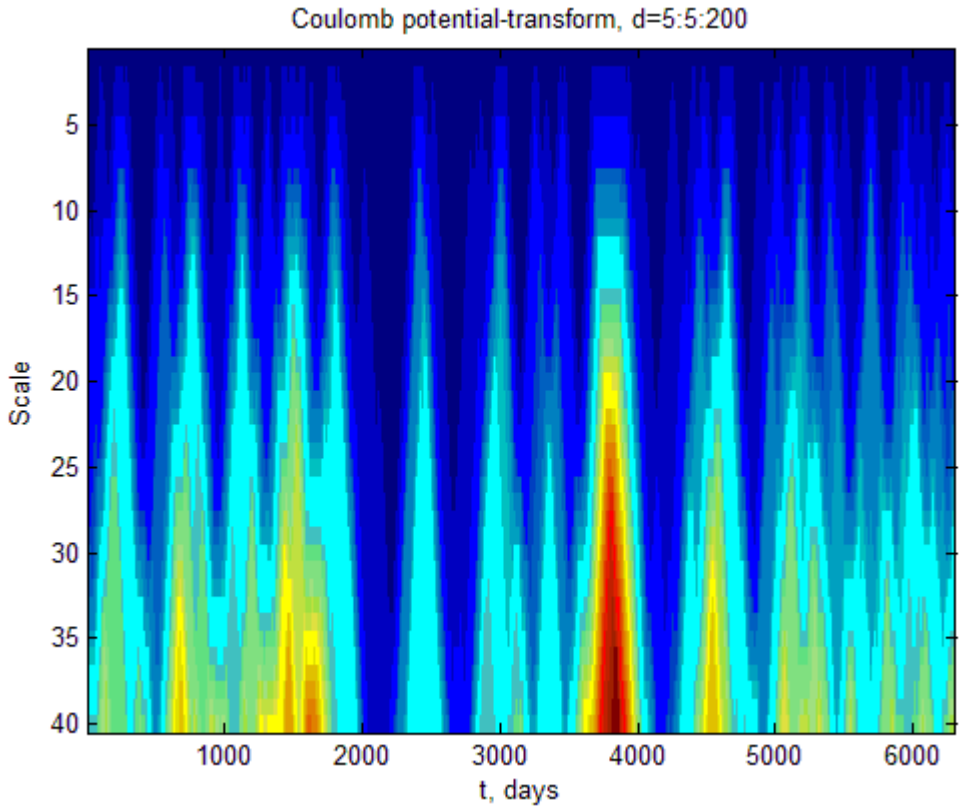
Fig.4. Potential curves for d = 200: a – Ising potential; b – Coulomb potential

As it is clear from Fig.4 the curves are very similar. Potential values, of course, are significantly different but both potentials equally well describe the specific features of time series dynamics. Small differences take place in the magnitude of the peaks which is may be due to the different scales of the curves.

Let us consider results of 1-D to 2-D potential transformation. Corresponding images are shown in Fig.5. For comparison in Fig.6 CWT is shown.



a



b

Fig.5. TS potential transformation: a – Ising potential; b – Coulomb potential

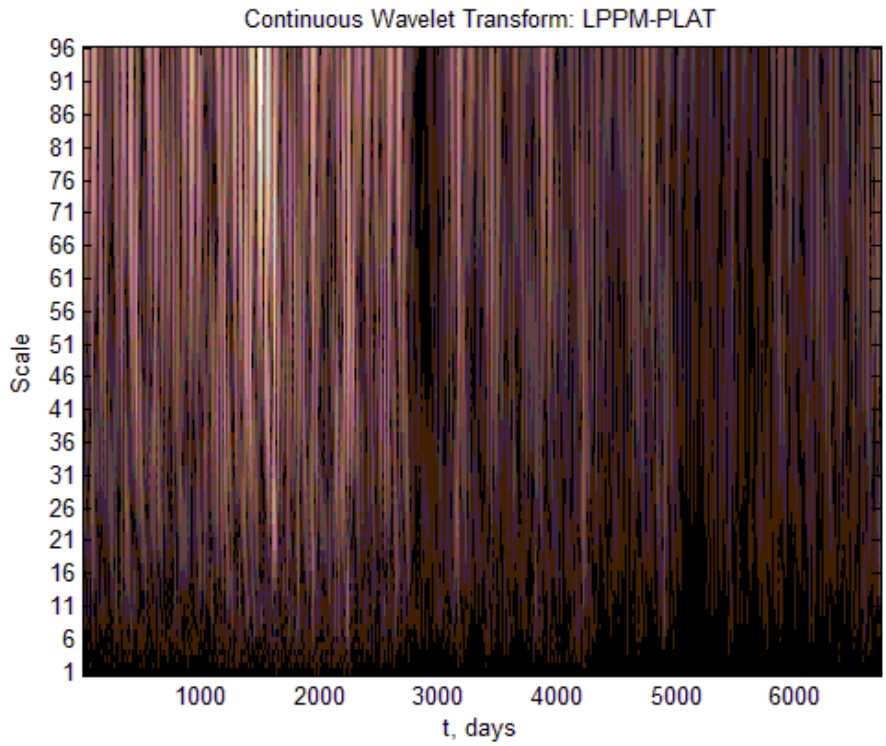


Fig.6. Continues wavelet transform of time series (Daubechies wavelet was used)

From the comparison of Fig.5 and Fig.6 follows that CWT do not show specific features of time series as clear as potential transformations. So, the potential transformation may be considered as effective additional tool to wavelet analysis.

Conclusion

New method of analysis of time series was proposed. The method uses potential transformation of time series. The essence of the potential transformation is the following. The first is the binarization of the time series. As a result, points are divided into two classes. Points of each class were considered as interacting pseudo particles. The potential is calculated for the interaction between points of one of the two classes. The potential was calculated using two interactions: Ising spin-spin interaction and Coulomb interaction. Moving window was used to calculate the potential value for every point of time series.

The results show:

1. Both potential transformations give similar results.

2. Specific points of the time series are better detected by a potential transformation than by using continues wavelet transform.

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