Model Order Reduction – Also with Many Inputs and Outputs?

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Abstract

Model order reduction has become an indispensable tool for multibody systems especially those including flexible bodies. The flexible bodies are typically discretized with the finite element method and mathematically described by their mass matrix, stiffness matrix, and possibly a damping matrix. For complex structures, these matrices may become very large. The model order reduction methods allow to systematically decrease the size of those system matrices by eliminating the properties of the system which have the least influence to the dynamic behavior of the system. Traditionally, this was done manually, e.g. by doing a modal analysis and ignoring high-frequency modes. However, better methods are now available based on system-theoretic features, e.g. an optimal approximation of the transfer behavior between system inputs and outputs.

One specific input-output based method is moment matching with Krylov subspaces. This method is based on the idea that the transfer function of the original model and the reduced model are matched at different frequency shifts. The choice of frequency shifts is of great importance. For example, in [2] an automated shift selection algorithm was proposed that guarantees that the approximation error of the reduced model does not exceed a predefined error bound, as shown in Fig. 1. This method is one way of obtaining a relatively small model with a small error.

Another possibility is to use a Greedy algorithm to select the frequencies with the largest error as next expansion points. To circumvent the computational complex calculation of the transfer function of the full order model, there are also methods available which provide error estimators, see e.g. [1]. Based on multiple different reduced order models, these methods give an accurate estimate of the transfer function error. In combination with the Greedy algorithm, these error estimators can efficiently be used to iteratively select the shifts.

One particular challenge with these methods is when the original system has many inputs or many outputs because the size of the reduced model depends directly on the number of inputs and outputs. This can lead to relatively large reduced models which goes against the purpose of model order reduction. For this reason, more sophisticated methods have to be used for this class of systems. For moment matching based method, this is often done by using tangential directions. These tangential directions reduce the number of inputs and outputs that are used to calculate the reduced subspace. However, the transfer function of the reduced order model does not match the transfer function of the full order model exactly, but only along the tangential direction.

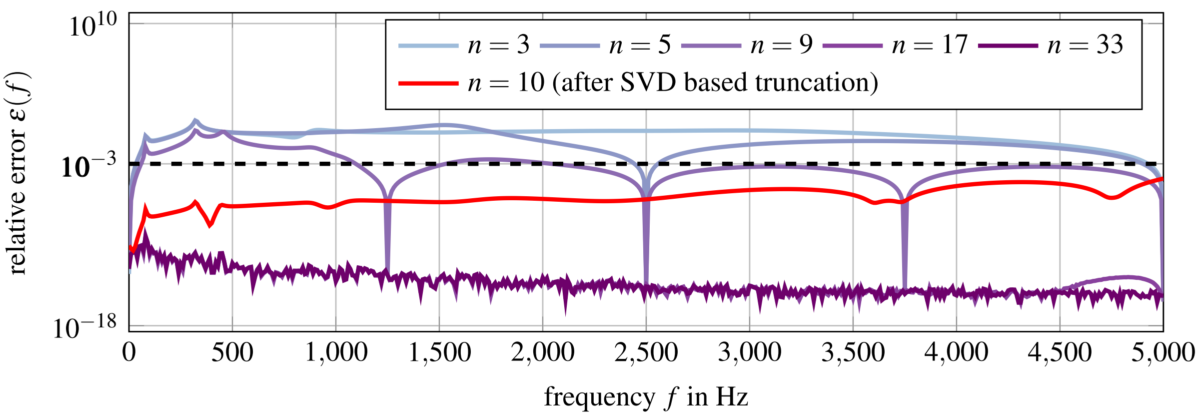

Figure 1: Error of reduced models computed with linear shift selection and SVD based truncation to satisfy predefined error bound.

In this talk it will be shown where challenges originate and what potential ideas there are to overcome these issues. There is a clear focus on linear systems, although also work has been spent on the reduction of nonlinear systems. Application examples of different complexity are utilized to demonstrate the demand and usefulness. An especially interesting use of many inputs and outputs originates from the coupling of different physical domains. One very complex system where this coupling is extremely relevant is a helicopter during flight. Here, reduced order models are utilized to compute the deformations of the helicopter cell, as shown in Fig. 2, that result from aerodynamic forces onto its surface.

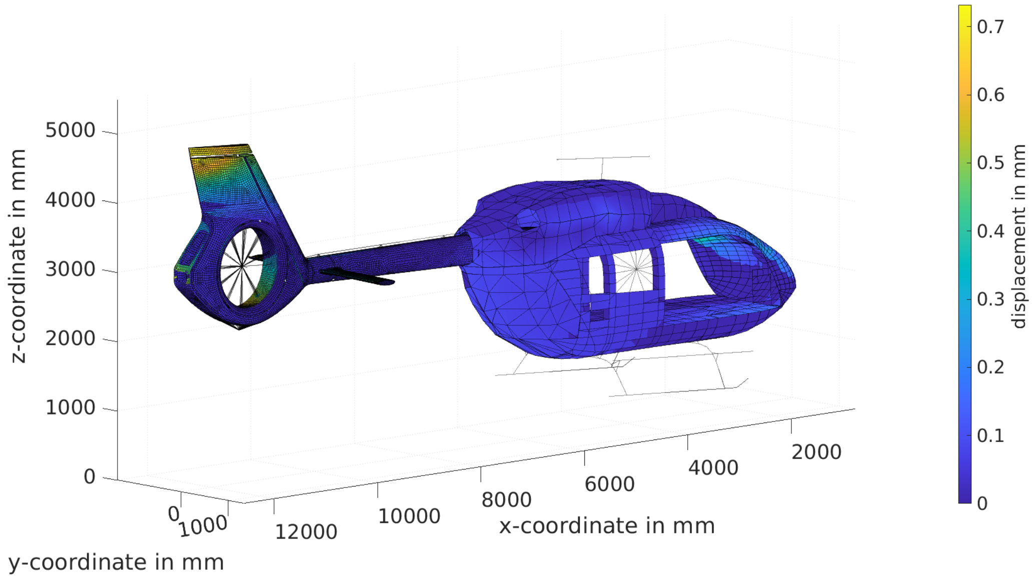

Figure 2: Deformation of the finite element model of a helicopter cell caused by aerodynamic surface loads resulting from a coupled simulation with fluid-structure based on a reduced order model.

References

[1] Feng, L.; and Benner, P.: On Error Estimation for Reduced-Order Modeling of Linear Non-Parametric and Parametric Systems. ESAIM: Mathematical Modelling and Numerical Analysis, Vol. 55, No. 2, pp. 561–594, 2021. [DOI: 10.1051/m2an/2021001]

[2] Frie, L.; Eberhard, P.: On shift selection for Krylov subspace based model order reduction. Multibody System Dynamics, Vol. 58, pp. 231–251, 2023. [DOI: 10.1007/s11044-022-09872-7]