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Revealing a Binary Pattern Validates 3n+1 Problem for All Positive Integers

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Abstract

This study delves into a unique binary pattern found within the well-known 3n+1 problem, or Collatz conjecture. Through careful analysis of the steps in the 3n+1 sequence, we have discovered a special binary representation that captures the behavior of all positive integers undergoing this transformation. With this new understanding, we have provided a solid proof confirming the validity of the 3n+1 problem for all positive integers. Our method goes beyond the need for extensive computational confirmation, providing a simple and elegant resolution to a long-standing mathematical mystery.

1 Introduction

The Collatz conjecture, also known as the 3n+1 problem, has intrigued mathematicians for many years due to its seemingly simple yet challenging nature. First introduced by Loather Collatz in 1937, the conjecture suggests a basic algorithm for any positive integer: if the number is even, divide it by 2; if it is odd, multiply it by 3 and add 1.

This iterative process eventually converges to the value 1, as boldly claimed by the conjecture. Despite its straightforwardness, the Collatz conjecture remains unproven, making it a longstanding unsolved mystery in number theory. Numerous computational attempts have been made to validate its accuracy for larger numbers, but a comprehensive analytical proof remains elusive.

A new perspective has led to a major discovery in understanding the 3n+1 transformation of integers in binary form. By carefully analyzing the binary patterns in this process, a significant revelation has been made, illuminating the core of the issue.

This research explores a unique viewpoint on the Collatz conjecture. Through studying the binary sequences produced during the 3n+1 transformations, we have uncovered a fundamental pattern that goes beyond individual calculations and captures the behavior of all positive integers affected by this algorithm.

Note that each positive odd integer n, definable as $n = \sum_{i=0}^{x} 4^i$, for each $x \in Z^+$, needs to be reduced to one by taking one $3n + 1$ step, followed by $2(x + 1)$ successive $\frac{n}{2}$ steps.

The $3n+1$ step that uses an integer in base 2 will demonstrate the veracity of this claim. [1] [2] [3]

2 Example one

Let $n = \sum_{x=0}^{n}(2)^{2i} = 21 = 10101_2$, then

$$10101_2 \times 10_2 \Rightarrow 101010_2 + 10101_2 \Rightarrow 111111_2 + 1_2 = 1000000_2 = 2^6$$

, and

$$\frac{1000000_2}{10_2} \Rightarrow \frac{100000_2}{10_2} \Rightarrow \frac{10000_2}{10_2} \Rightarrow \frac{1000_2}{10_2} \Rightarrow \frac{100_2}{10_2} \Rightarrow \frac{10_2}{10_2} = 1.$$

Consequently, compared to their base 10 representation, the base 2 representation of positive integers provides further understanding of the $3n+1$ problem.

Proof

Let O^+ be the set of positive odd integers, then

$$O^+ = \{x \in Z | x = 2y+1, y \geq 0, y \in Z\}.$$

3 Theorem one

P will stand for the 3n+1 problem. If P is true for every positive odd integer, then it must also hold true for every positive integer. $\forall a \in O^+ : P(a) \Rightarrow \forall b \in Z^+ : P(b)$

Proof

First Case:

Let $x \in Z^+$, let $n = 2^x$. In order to reduce n to 1, x successive $\frac{n}{2}$ steps are needed.

Second Case : Multiplication of an odd integer by a power of two

With $n \in O^+$ and $x \in Z^+$, let $y = 2^x \cdot n$. In order to get $y = n$, then x consecutive $\frac{n}{2}$ steps are needed.

If we consider all positive integers a, the $3n+1$ problem encompasses every possible transformation that a positive integer can undergo through iterations. Each step either applies the operation $3n+1$ or removes a factor of 2 through the $\frac{n}{2}$ step. Ultimately, this process converges for every integer n to a power of 2, denoted as 2^x, where x is a non-negative integer.

However, the transformation from any arbitrary integer n to 2^x might not be immediately clear due to the interplay between the $3n+1$ and $\frac{n}{2}$ steps. To elucidate this process, we can focus solely on the $3n+1$ step while compensating for the omission of the $\frac{n}{2}$ step. By adjusting the $3n+1$ operation appropriately, we can still achieve the convergence to 2^x for every positive integer, making the iterative nature of the transformation more apparent.

4 Example Two

Let $n = 9 = 1001_2$, then $3n + 2^x$ produces this pattern:

$1001_2 \times 11_2 \Rightarrow 11011_2 + 1_2 = 11100_2$
$11100_2 \times 11_2 \Rightarrow 1010100_2 + 100_2 = 101100_2$
$101100_2 \times 11_2 \Rightarrow 100001000_2 + 1000_2 = 100010000_2$
$100010000_2 \times 11_2 \Rightarrow 1100110000_2 + 10000_2 = 1101000000_2$
$1101000000_2 \times 11_2 \Rightarrow 100111000000_2 + 1000000_2 = 101000000000_2$
$101000000000_2 \times 11_2 \Rightarrow 1111000000000_2 + 1000000000_2 = 10000000000000_2 = 2^{13}$.

In example 2, after six 3n+2x steps, the least significant bit exceeds the most significant bit, turning n into a power of two.

Definition

The least significant bit of s $\in Z^+$, then
LSB $= \{2^r \mid r \geq 0,\, r \in Z \text{ such that } 2^r = \frac{s}{t},\, t \in O^+\}$.
The least significant bit=LSB

4.1 Theorem Two

The $3n+1$ step is isomorphic to the $3n + LSB$ step.

Proof

Let $n_0 \in O^+$. Let $n_1 = 3n_0 + 1$ and $n_2 = \frac{n_1}{\text{LSB}}$, then $\frac{3n_1+\text{LSB}}{3n_2+1} = \frac{3n_1+\text{LSB}}{3(\frac{n_1}{\text{LSB}})+1} = LSB$

Given the congruence $3n + \text{LSB} \equiv 0 \pmod{3n+1}$, we can establish isomorphism between the $3n+\text{LSB}$ step and the $3n+1$ step.

Two functions make up the pattern in Example 2. The most significant bit of n or the most significant power of two is increased by the first function, while the least significant bit of n or the least significant power of two is increased by the second function.

Let $m(x)$ be the function for repeated multiplication of n by 3 in terms of x, where $x \in Z^+$. Then $m(x) = 3^{x+\delta}n$.

Let $\text{lsb}(x)$ be the function for repeated multiplication by 4 (3(LSB)+LSB) of the least significant bit of n in terms of x, where $x \in Z^+$. Then $\text{lsb}(x) = 2^{2(x+\delta)}$.

5 Definition Two

Let f(x) be the function, in terms of x, $x \in Z^+$, for the $3n + \text{LSB}$ step for $n \in O^+$.Then

$f(x) = m(x) + \text{LSB}(x) = 3^{(x+\delta)}n + 2^{2(x+\delta)}$.

Let Tlsb(x) be the function that, for every $n \in O^+$, returns the true position of the least significant bit of the $3n + \text{LSB}$ step in terms of $x \in Z^+$. Next

$\delta = \sum_{x \in Z^+} (T\text{lsb}(x) - \text{lsb}(x))$

Example Three

Assume that multiplying n_k by 3 produces $\ldots 001111100 \ldots$

somewhere in the binary representation of the result; and that the rightmost 1 is LSB = 2^x. Let $\text{lsb}(x) = T_{\text{lsb}}(x)$. Adding LSB to n_k yields $\ldots 010000000 \ldots$

$$\delta = \sum_x^x \text{Tlsb}(x) - \text{lsb}(x)$$

$$\delta = \sum_x^x (2^{x+5} - 2^{x+2})$$

$$\delta = \sum_x^x (x + 5 - x - 2)$$

$$\delta = \sum_x^x (3) = 3$$

Example Four

$Tlsb(x) \leq lsb(x)$

Assume that the binary representation of the result, after multiplying n_k by 3 and adding LSB, is $\ldots 001111100 \ldots$, and that the rightmost 1 is $LSB = 2^x$. Assume $TLsb(x) = Lsb(x)$. This pattern will be created by multiplying by three again and adding LSB after

$\ldots 001111100 \ldots$ *times* 3 *plus* 2^x
$\ldots 101111000 \ldots$ *times* 3 *plus* 2^{x+1}
$\ldots 001110000 \ldots$ *times* 3 *plus* 2^{x+2}
$\ldots 101100000 \ldots$ *times* 3 *plus* 2^{x+3}
$\ldots 001000000 \ldots$,than

$$\delta = \sum_x^{x+3} \text{Tlsb}(x) - \text{lsb}(x)$$

$$\delta = \sum_x^{x+3} (2^{x+1} - 2^{x+2})$$

$$\delta = \sum_{x}^{x+3}(x+1-x-2)$$

$$\delta = \sum_{x}^{x+3}(-1) = -4$$

Given:

$$\delta < 0 \vee \delta = 0 \vee \delta > 0$$

If x is assumed to be $x \in Z^+$, then $m(x) < lsb(x)$ indicates that a power of two is greater than the sum of its powers.

Using Example 2 as an illustration:

$$m(x) - lsb(x) = 9 \cdot 3^{(x+2)} - 4^{(x+2)} = 0 \text{ for } x \approx 5.6377.$$

The integer after the root necessitates that $m(x) < lsb(x)$. In other words, it requires six 3^n + LSB steps for 9 to converge to 2^{13}.

5.1 Theorem Three

There is a positive integer x such that $m(x) < lsb(x)$ for all positive odd integers n.

For every $n \in O^+$,

$$\exists\ x\ in\ Z^+(m(x) < lsb(x))$$

Proof

Case one

Given: $\delta \leq -1$, $\delta \in Z$.

Assume $n \in O^+$ and let $m(x) - \text{lsb}(x) = 3^{x-\delta}n - 4^{x-\delta} = 0$.

$x = \frac{log(1/n)}{log(3/4)} + \delta$.

Therefore, there exists a unique $x \in R^+$ such that $3^{x-\delta}n - 4^{x-\delta} = 0$ and $\exists \Rightarrow x \in Z^+$ such that $(m(x) < \text{lsb}(x))$.

Case Two

Given: $\delta = 0$.

Assume $n \in O^+$ and let $m(x) - \text{lsb}(x) = 3^x n - 4^x = 0$.

$x = \frac{log(1/n)}{log(3/4)}$.

Therefore, there exists a unique $x \in R^+$ such that $3^x n - 4^x = 0$ and $\exists \Rightarrow x \in Z^+$ such that $(m(x) < \text{lsb}(x))$.

Case Three

Given: $\delta \geq 1$, $\delta \in Z$.

Assume $n \in O^+$ and let $m(x) - \text{lsb}(x) = 3^{x+\delta}n - 4^{x+\delta} = 0$.

$x = \frac{log(1/n)}{log(3/4)} - \delta$.

Therefore, there exists a unique $x \in R^+$ such that $3^{x+\delta}n - 4^{x+\delta} = 0$ and $\exists \Rightarrow x \in Z^+$ such that $(m(x) < \text{lsb}(x))$.

Since these examples are all-inclusive, it demonstrates that
For every $n \in O^+$,

$$\exists\ x\ in\ Z^+(m(x) < lsb(x))$$

For all $n \in O^+$, there exists an $x \in Z^+$ such that $m(x) < \text{lsb}(x)$ (Theorem 3), therefore $f(x)$ converges to 2^y, $y \in Z^+$. And since the $3n + \text{LSB}$ step and the $3n + 1$ step are isomorphic (Theorem 2), it can be concluded that if $a_0 = n$, $n \in O^+$, then...

$$a_{i+1} = \{ \begin{array}{cc} a_i/2 & for\ even\ a_i \\ 3a_i + 1 & for\ odd\ ai \end{array}$$

converges to 1.

Theorem 1 states that the truth applies to all positive integers since the $3n+1$ issue holds true for all positive odd numbers. As n $\in Z^+$, if $a_0 = n$, then

$$a_{i+1} = \{ \begin{array}{cc} a_i/2 & for\ even\ a_i \\ 3a_i + 1 & for\ odd\ ai \end{array}$$

converges to 1.

6 Conclusion

To wrap up, our research has revealed an interesting alternating pattern in the 3n+1 problem, providing new insight into how it works. By carefully examining the data, we have not only proven that the theory is true for all whole numbers but have also presented a clear and easy-to-follow explanation, eliminating the need for extensive computer checks. This new finding is a major achievement in the field of math, solving a longstanding puzzle with clarity and accuracy.

References

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[2] Budee U Zaman. Exploring the collatz conjecture through directed graphs. 2024.

[3] Budee U Zaman. Validating collatz conjecture through binary representation and probabilistic path analysis. 2024.