Modeling the Processes with Almost Cyclostationary Structure

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Abstract. This paper is devoted to establish a computational approach to predict the processes with almost cyclostationary structure. The main idea is based on the estimating of the support of spectra and using the discrete Fourier transform and periodogram of almost cyclostationary processes. The simulated and real datasets are applied to study the performance of the introduced approach. The results confirm that the presented method acts efficiently for simulated and real datasets.

Keywords: Almost Cyclostationary, Almost Periodically Correlated, Discrete Fourier Transform, Periodogram, Prediction, Spectral Analysis.

1. Introduction

Stationarity is an essential assumption in classical time series modeling. This assumption is not satisfied in many datasets, specially when these have periodic rhythm. In these situations, cyclostationary (CS) and almost cyclostationary (ACS) processes are naturally applied to model the rhythmic component. The ACS is a large non-stationary time series class that contained stationary and CS processes. The mean and auto-covariance functions of ACS are almost periodic. The spectra of these processes are supported on lines that are parallel to the main diagonal, $T_j(x) = x \pm \alpha_j, j = 1, 2, \ldots,$ in spectral square $[0,2\pi) \times [0,2\pi)$. The theories and applications of CS and ACS time series have been studied in many references such as Gladyshev [(1961); (1963)], Gardner (1991), Hurd (1991), Hurd and Leskow (1992), Leskow and Weron (1992), Gardner (1994), Leskow (1994), Lii and Rosenblatt [(2002); (2006)], Gardner et al. (2006), Hurd and Miamee (2007), Lenart [(2008); (2011)], Napolitano (2012), Lenart (2013), Lenart and Pipien [(2013a); (2013b)], Mahmoudi et al. (2015), Napolitano [(2016a); (2016b)], Mahmoudi and Maleki (2017), Nematollahi et al. (2017), Lenart and Pipien (2017), and Mahmoudi et al. [(2018a), (2018b), (2018c)].

Definition 1: Almost Periodic Function [Corduneanu (1989)]

A function $f\ (t) :\ Z\ \rightarrow\ R$ is said to be almost periodic in $t\ \in\ Z$ if for any $\varepsilon > 0$, there exists an integer $L_\varepsilon > 0$ such that among any $L_\varepsilon > 0$ consecutive integers there is an integer $p_\varepsilon > 0$ such that

$$sup_{t \in \mathrm{Z}} |f(t + p_\varepsilon) - f(t)| < \varepsilon.$$

Definition 2: ACS Process [Mahmoudi et al. (2018a)]

A second order process $\{X_t : t\ \in\ Z\}$ is called ACS if the process has almost periodic mean, $\mu(t) =\ E(X_t)$, and autocovariance, $B(t, \tau) =\ cov(X_t, X_{t+\tau})$, at t, for every $\tau \in$ Z.

As Mahmoudi et al. (2018a), the following assumptions have been considered in this work:

(A1) $\{X_t : t\ \in\ Z\}$ *is a zero-mean and real-valued time series.*

(A2) X_t *is an ACS process.*

By these assumptions, the autocovariance function $B(t, \tau)$ can be represented by

$$B(t,\tau) \sim \sum_{\omega \in W_t} a(\omega,\tau) e^{i\omega t},$$

where

$$a(\omega,\tau) = \lim_{n\to\infty} \left(\frac{1}{n} \sum_{j=1}^{n} B(j,\tau)\, e^{-i\omega t} \right),$$

and for fixed τ. Also as Corduneanu (1989) and Hurd (1991) indicated the set $W_\tau = \{\omega \in [0,2\pi): a(\omega,\tau) \neq 0\}$ is a countable set of frequencies.

(A3) $W = \bigcup_{\tau \in Z} W_\tau$, *is a finite set and the spectra of* X_t *is supported on lines that are parallel to the main diagonal,* $T_j(x) = x \pm \alpha_j, j = 1, 2, \ldots$, *in spectral square* $[0,2\pi) \times [0,2\pi)$. Thus we have

$$B(t,\tau) = \sum_{\omega \in W} a(\omega,\tau) e^{i\omega t},$$

and the spectral measure of X_t, will be supported on the set

$$S = \bigcup_{\omega \in W} \{(\nu,\gamma) \in [0,2\pi)^2 : \gamma = \nu - \omega\}.$$

Moreover, the coefficients

$$a(\omega,\tau) = \int_0^{2\pi} e^{i\xi\tau} r_\omega(d\xi),$$

are the Fourier transforms of the measures $r_\omega(\cdot)$.

We note that the r_ω will be identified if the spectral measure of X_t be restricted on the line $\gamma = \nu - \omega$, modulo 2π, where $\omega \in W$.

Remark: In the rest of paper, all equalities of frequencies are modulo 2π.

(A4) r_0 *is an absolute continuous measure with respect to the Lebesgue measure.*

Dehay and Hurd (1994) shown by considering this assumption and $\sum_{\tau=-\infty}^{\infty} |a(\omega,\tau)| < \infty$, for any $\omega \in W$, result in a spectral density function $f_\omega(\cdot)$ exists such that

$$f_\omega(\nu) = \frac{1}{2\pi}\sum_{\tau=-\infty}^{\infty} a(\omega,\tau)e^{-i\nu\tau}.$$

Consequently, an ACS process with support on a finite number of cyclic frequencies is represented by

$$X_t = \int_0^{2\pi} e^{-itx}\zeta(dx), \;\; t \in \mathbb{Z},$$

where ζ is a random spectral measure on $[0, 2\pi)$ such that

$$E\left(\zeta(d\theta)\overline{\zeta(d\theta')}\right) = 0, (\theta,\theta') \notin S.$$

As Mahmoudi et al. (2018a) indicated, the spectral distribution and density matrices of ζ, are defined by

$$\boldsymbol{F}(d\lambda) = \left[F_{k,j}(d\lambda)\right]_{j,k=1,\dots,m},$$

and

$$\boldsymbol{f}(\lambda) = \frac{d\boldsymbol{F}}{d\lambda} = \left[f_{k,j}(\lambda)\right]_{j,k=1,\dots,m},$$

respectively, where

$$F_{k,j}(d\lambda) = E\left(\zeta\,(d\lambda+\alpha_k)\overline{\zeta\,(d\lambda+\alpha_j)}\right), k,j = \, 1,\dots,m,$$

and $f_{k,j}$ is spectral density correspond to $F_{k,j}$.

Definition 3: Discrete Fourier Transform (DFT)

Assume $X_0, \dots, X_{N-1}$, are a sample of size N from ACS process $\{X_t : t \in Z\}$. The DFT of the this sample is defined by

$$d_X(\lambda) = N^{-1/2}\sum_{t=0}^{N-1} X_t e^{-it\lambda} \;, \lambda \in [0,2\pi).$$

Definition 4: Periodogram

Assume a sample $X_0, \ldots, X_{N-1}$, from ACS process $\{X_t: t \in Z\}$. The periodogram of the finite sequence $X_0, \ldots, X_{N-1}$, is defined by

$$I_X(\lambda) = |d_X(\lambda)|^2, \lambda \in [0,2\pi).$$

The distribution of DFT and periodogram of ACS processes are widely studied by Lenart (2013), Lenart and Pipien [(2013a); (2017)] and Mahmoudi et al. (2018a).

The aim of this paper is to establish a computational approach to predict the processes with almost cyclostationary structure. The main idea is based on the estimating of the support of spectra and using the discrete Fourier transform and periodogram of ACS processes. In Section 2, prediction problem for ACS time series is studied. The ability of the introduced approach is also studied using simulation study and real data analysis, in Section 3.

2. Prediction of ACS Processes

Let $\{X_t, t \in \mathbb{Z}\}$ be ACS process with spectral density $\mathbf{f}(\lambda), \lambda \in [0,2\pi)$. The supports of the spectra for ACS processes are the lines $T_j\left(T_k^{-1}(x)\right)$, where $T_j(x): B_1 \longrightarrow B_j$, is defined by $T_j(x) = x + \alpha_j$, for $j = 1, \ldots, m$.

Soltani and Parvardeh (2006) showed the best predictor for $X_{t+\tau}, \tau > 0$, is given by

$$\hat{X}(t+\tau) = \sum_{k=1}^{m} \hat{X}_k(t+\tau),$$

where

$$\hat{X}_k(t) = \sum_{l=-\infty}^{+\infty} (\hat{g}_{t,k})(l) Z_{k,l},$$

$$\hat{g}_{t,k}(x) = \sum_{j=1}^{m} e^{itT_j(x)} a_{jk}(x),$$

and $\{Z_{k,l}\}$, $k = 1, \dots, m$ are orthogonal white noise series. Also $a_{jk}(x), j, k = 1, \dots, m$, are the components of Cholesky decomposition of spectral density **f**, given by

$$\mathbf{f}(x) = \mathbf{A}(x)\mathbf{A}^*(x),$$

where $\mathbf{A}^*$ is conjugate transpose of **A**.

In real problems, $T_j(x) = x + \alpha_j$, $j = 1, \dots, m$, and $a_{jk}(x), j, k = 1, \dots, m$, are unknown. Mahmoudi et al. (2018a) applied the following procedures to estimate these unknown functions.

2.1. Procedure for Estimating $T_j, j = 1, \dots, m$

Let

$$\hat{C}(\lambda, \lambda') = Corr(|d_X(\lambda)|, |d_X(\lambda')|).$$

The summary of estimation procedure of T_j's is as follows:

(i) For given $\lambda \in [0,2\pi)$, apply the moving block bootstrap (MBB) methodology to produce *n* sample of $d_X(\lambda)$.

(ii) For $\lambda \in B_1$ and $\lambda' \in B_j, j = 1, \dots, m$, calculate $\hat{C}(\lambda, \lambda')$ using n samples of DFT in λ and λ', $\{d_1(\lambda), \dots, d_n(\lambda)\}$ and $\{d_1(\lambda'), \dots, d_n(\lambda')\}$.

(iii) Fix $\lambda \in B_1$ to obtain $\lambda^* \in B_j$ such that (λ, λ^*) maximizes $\hat{C}(\lambda, \lambda')$.

(iv) Repeat Step (iii) until finding $\lambda_1^*, \dots, \lambda_J^* \in B_j$ corresponding to $\lambda_1, \dots, \lambda_J \in B_1$.

(v) Assign $\lambda_k^* = \hat{T}_j(\lambda_k), k = 1, \dots, m$; which estimate $T_j, j = 1, \dots, m$.

2.2. Procedure for Estimating $a_{jk}(x), j, k = 1, \dots, m$

Mahmoudi et al. (2018a) defined the periodogram of the finite ACS time series as

$$\boldsymbol{I}_X^m(\lambda) = \boldsymbol{d}_X^m(\lambda)\boldsymbol{d}_X^{m*}(\lambda),$$

where

$$\boldsymbol{d}_X^m(\lambda) = \left(d_X(T_1(\lambda)), d_X(T_2(\lambda)) \dots, d_X(T_m(\lambda))\right)' , \lambda \in B_1.$$

They showed that $\hat{\mathbf{f}}(\lambda) = \frac{\mathbf{I}_X^m(\lambda)}{2\pi}$, is an asymptotically unbiased estimator for $\mathbf{f}(\lambda)$, $\lambda \in B_1$. Therefore we can estimate $\mathbf{A}(\mathrm{x})$ by the square root of $\hat{\mathbf{f}}(\mathrm{x})$, i. e.,

$$\widehat{\mathbf{A}}(\mathrm{x}) = \mathbf{sqrt}\left(\hat{\mathbf{f}}(\mathrm{x})\right).$$

3. Simulation Study

In this section, first we demonstrate the simulation results of using the presented method in spectral support estimation. Then the applicability of the method is evaluated by a real example.

3.1. Simulation Study

To analyze the ability of the presented approach, different datasets are generated from the process

$$X_t = (1 + \cos(\omega t))Y_t, \omega \in (0, \infty),$$

where

$$Y_t = Z_t + 0.5Z_{t-1},$$

and Z_t is a sequence of IIDN(0,1).

The simulations are accomplished after 1000 runs and using the R 3.3.2. software (*R Development Core Team, 2017*).

The spectral mass of X_t is supported on the lines given by

$$T_1(x) = x, T_2(x) = x + \omega, T_3(x) = x - \omega, T_4(x) = x - 2\omega, T_5(x) = x + 2\omega.$$

Figure 1 shows the spectral square$[0,2\pi)^2$, for

$$\omega = \{0.5,1,2\}.$$

The results of the estimation procedure are shown in Table 1.

The presented approach is employed to predict 10 last observations $(\hat{X}_{N-9}, \dots, \hat{X}_N,)$ based on $\{X_1, \dots, X_{N-10}\}$. The results are summarized in Table 1. The first and second columns report the values of the mean absolute error (MAE) and the mean square error (MSE), which is respectively presented by

$$MAE = \frac{1}{10000} \sum_{k=1}^{1000} \sum_{j=N-9}^{N} \left|\hat{X}_{k,j} - X_{k,j}\right|,$$

and

$$MSE = \frac{1}{10000} \sum_{k=1}^{1000} \sum_{j=N-9}^{N} \left|\hat{X}_{k,j} - X_{k,j}\right|^2,$$

where $X_{k,j}$ and $\hat{X}_{k,j}$ are the real and predicted values for X_j in replication k.

As Table 1 indicates, the values of MAE and MSE are very close to zero and consequently we can accept that the introduced approach acts well, especially as N is large.

3.2. Real data

Now, we illustrate a real example to show the ability of the introduced approach in the real world applications. The dataset includes the first difference of centered moving average filter 2×12 moving average (MA) applied for logarithm of industrial production index (IPI) in Poland (2005 = 100%) since January 1995 untile December 2009, Lenart and Pipien (2013b). Figure 2 shows the IPI, the first difference of centered moving average filter 2×12 MA applied for IPI and the first difference of centered moving average filter 2×12 MA applied for logarithm of IPI, respectively. Lenart and Pipien (2013b) detected an ACS time series with spectra on the lines $T_j(x) = x \pm \alpha, \alpha \in \{0.062, 0.153, 0.258\}$. Figure 3 also shows the spectral coherency graph. This graph also reveals that the considered ACS time series by Lenart and Pipien (2013b) can be a good choice to fit dataset.

The presented method is applied to predict 10 last observations $(\hat{X}_{N-9}, \ldots, \hat{X}_N,)$ based on $\{X_1, \ldots, X_{N-10}\}$. The results are summarized in Table 2. The columns show the values of absolute error (AE) and square error (SE), which is respectively defined by

$$AE = |\hat{X}_j - X_j|,$$

and

$$MSE = \frac{1}{10}|\hat{X}_j - X_j|^2,$$

where X_j and $\hat{X}_j$ are the real and predicted values for X_j.

As Table 2 indicates, the values of AE and SE are very close to zero and consequently we can accept that the presented approach acts well in real world problems.

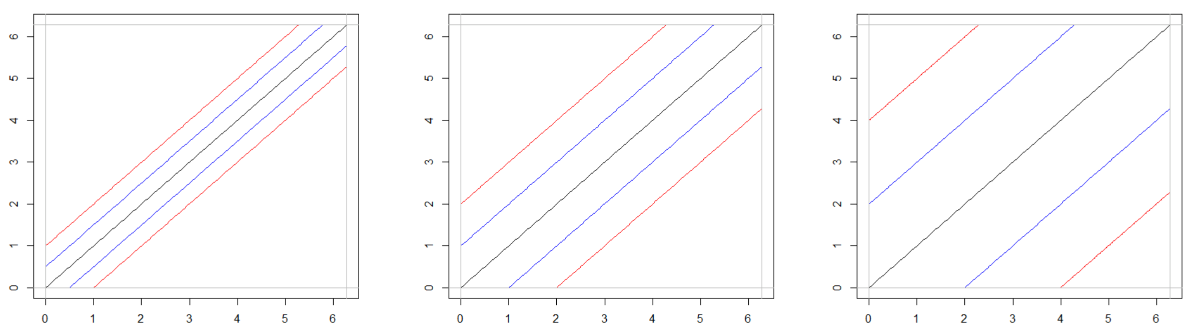

Figure 1: Spectral square, Left: $\omega = 0.5$, Middle: $\omega = 1$, and Right: $\omega = 2$

Table 1: The values of MAE and MSE for simulated datasets

N	MAE	MSE
100	0.002942547	8.32855E-06
200	0.002929578	7.61992E-06
500	0.002695950	8.91804E-06
1000	0.002640224	8.49954E-06
5000	0.002199655	6.57345E-06
10000	0.002023741	6.95971E-06

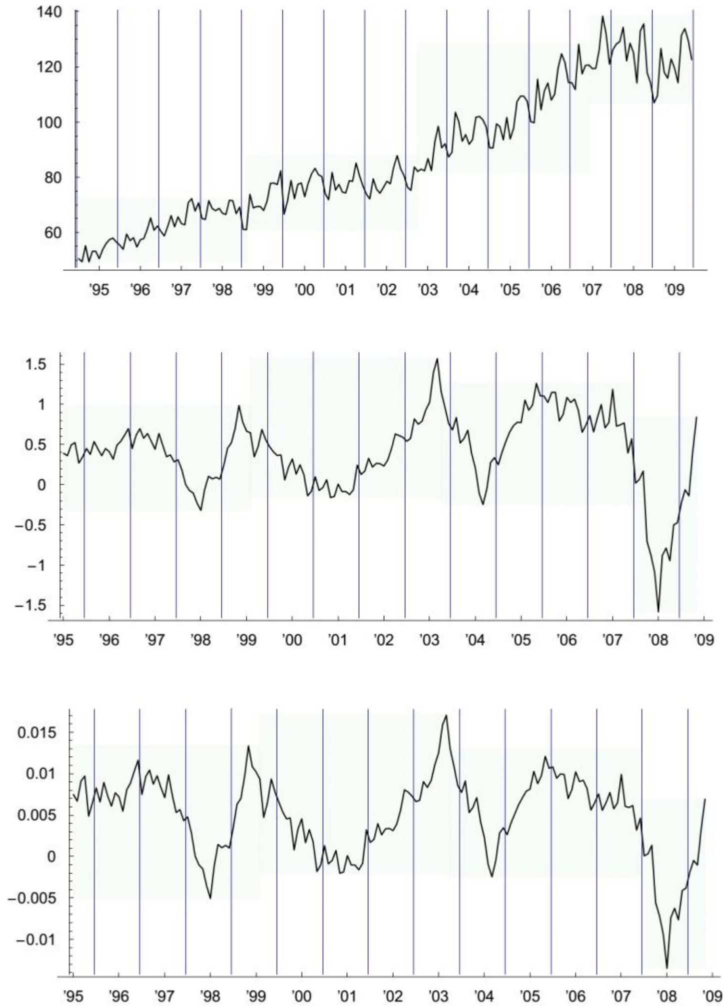

Figure 2: The IPI (Top), the first difference of centered moving average filter 2×12 MA applied for IPI (Middle) and the first difference of centered moving average filter 2×12 MA applied for logarithm of IPI (Bottom) in Poland (2005 = 100%) since January 1995 untile December 2009

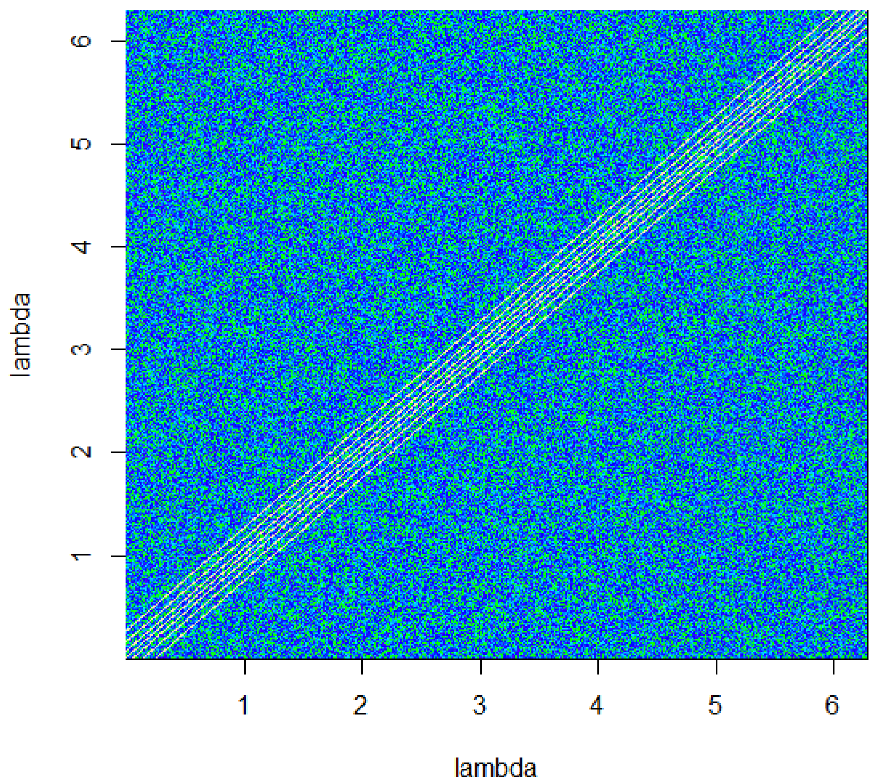

Figure 3: Spectral coherency graph for the first difference of centered moving average filter 2×12 MA applied for logarithm of IPI in Poland (2005 = 100%) since January 1995 untile December 2009

Table 2: The values of AE and SE for real dataset

j	AE	SE
N-9	0.002196039	4.47264E-06
N-8	0.002674178	5.68371E-06
N-7	0.002453326	4.54062E-06
N-6	0.002992652	5.47701E-06
N-5	0.002766690	4.07448E-06
N-4	0.002340976	5.40244E-06
N-3	0.002684742	7.07334E-06
N-2	0.002285459	8.97162E-06
N-1	0.002690938	4.07284E-06
N	0.002334971	8.91985E-06

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