Modeling of capital investments public financing volume using logistic curves

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ABSTRACT
A methodological justification of the threshold value of the public financing of capital investments as a point of reverse is provided within the current research (LPI). The point of reverse means a share of public expenditures in the structure of GDP, which excess will slow down economic growth. The authors propose the solution of the scientific problem that considers a point of reverse calculation for the Ukrainian financial system. The LPI threshold method is based on the construction of mathematical functions used to model and predict S-shaped processes, namely: the modified exponent, the Pearl and Reid logistic curve and the Gompertz function. According to the empirical studies' results in Ukraine, the modified exponent is showing the most adequate results in the verification of the criteria fulfillment (Adjusted Multiple Determination Ratio, Akaike's Information Criterion, Bayesian Information Criterion). The upper threshold of public capital expenditures in Ukraine (LPI) is 7% of GDP, while the national economy growth in the form of GDP growth will not be possible in case of public capital investments reduction to 0.86%. The analysis shows that the amount of capital expenditures from the state budget is an indicator of the national financial policy that facilitates the extention of economic agreements or their restriction due to the multiplier effect. Thereby, public investment should have limited and mobile measure. At the same time, they should not be a substitute for all existing sources of investment activity financing.
Keywords: *public finances, economic growth, economic reverse, public expenditure, capital investment, public investment, financial policy, financial resources, modified exponent, Perl and Reid logistic curve, Gompertz function*

1. INTRODUCTION

The transformation processes dynamics within the economic system, increasing dependence of its development on the challenges of internal and external environment actualize the issue of strengthening the state role in the structural correction of the economy and ensuring the foundations of sustainable development on the national and regional level.

The effectiveness of structural and technological renewal, including innovation and investment modernization of production, primarily depends on sufficient resources for these processes and the public role in its formation. It is known that the conceptual foundations of public financial policy, including investment, depend on the type of economic model and the role of the state in social and economic relations. Public capital expenditures can both stimulate economic growth and provide a damaging effect on market agents (reducing private sector savings through fiscal pressure or reducing free financial resources in the market through a "crowding-out effect", which slows down capital accumulation in private sectors in the long run). In such circumstances, it is an important task for public financial policy to determine the target volume of public capital expenditures (public investment) that would ensure capital accumulation and GDP growth in the long term.

2. REVIEW OF RELATED LITERATURE

The research of most scholars in the field of public finance is mainly concerned with the features, trends, optimal size of public spending as a whole, without substantiating the size and impact of public capital expenditures or government investment on the economic development. Thus, O. Vasilik [1], V. Oparin [2], V. Fedosov, P. Yukhimenko [3] studied the functions of the state budget, but did not determine the quantitative parameters of public expenditures and their distribution between individual units. V.Zimovets [4] investigated the problems of forming the state financial policy of economic development and proposed a theoretical approach to determining the optimal size of public spending in a corrupt extraction economy.

D. De Avila and R. Strauch [5], V. Tanzi and L. Schuknecht [6], as well as M. Connolly and C. Li [7] empirically established the relationship between public spending and the economic development of OECD countries. D. Lupu, M. Petrisor, A. Bercu and M. Tofan [8] considered the correlation between real GDP growth and 10 different categories of public expenditures, according to their functional classification, for 10 selected Central and Eastern European countries (using an autoregressive-distributed lag model (ARDL)). E. Hsieh and K. S. Lai [9] take into account Barro's endogenous growth model and make attempts to reveal the nature of the relationship between government expenditures and economic growth by examining the intertemporal interactions between the growth of a real GDP per capita, the share of government spending, and the ratio of private investment to GDP of the G-7countries. A. Illarionov and N. Pivovarova [10] calculated the marginal level of government expenditures in GDP for the post-Soviet countries. R. Sánchez [11] analysed the optimal structure of public expenditures and their distribution between public consumption and public investment.

We should also mention the authors who researched the impact of public investment on real GDP. Their studies are grouped according to the evaluation methods they used: a vector autoregression (A. Pereira and O. Roca-Sagales [12], J. Creel and G. Poilon [13]) and the vector model of error correction (G. Everaert [14], A. Pereira and J. Andraz [15], M. Ramirez [16], A. Pina and M. Aubyn [17], C. Annala, R. Batina and J. Feehan [18]). It should be noted that A. De la Fuente establishes a limit on public investment for developed countries at 2% of GDP [19].

At the same time, the latest analysis of OECD.Stat has shown that the share of government spending on financing the investment needs of the economy in developed countries ranges from almost 9% of GDP in Greece to 2% in Israel. Besides, industrial countries have outrun all other highly developed countries in this ratio, which is related to the need of production modernization and the need of resource consolidation for its implementation.

Therefore, an important theoretical and practical objectives iare to determine an adequate level of public capital expenditures, which will have a stimulating effect on GDP growth, private investment, and the accumulation of productive capital in the non-governmental sectors in the long run.

3. RESEARCH PURPOSE

The research aims to determine the threshold for public financing of capital investment as a point of reverse, namely a share of public expenditure in GDP. The economic growth will slow down in case of the above-mentioned share exceeding in a particular timeframe, and also calculate this indicator for the financial system of Ukraine.

4. THE MAIN RESULTS OF THE STUDY

4.1. Theoretical background

The threshold of capital investments public financing is determined using the logistic curves construction. Logistic curves or S-curves are not selected as the current research instrument by accident. The above-mentioned curves are the best instruments to describe the exponencial processes, when the growth depends on the already reached level rather than on various restrictions. In fact, S-curves describe two sequential processes: one process is connected with acceleration of development, the other one is connected with its deceleration. The inflection point determines its culminating moment, while asymptote - the limit of the process development. For instance, there are at least three mathematical functions that can be used to model and predict S-shaped processes. In particular, the modified exponential, the logistic curve (Pearl and Reed) and Gompertz function are among them [20].

The modified exponent differs from the simple exponent due to its additional component - the asymptote K (it is the upper asymptote in our case):

$$Y_t = K - ab^t, \tag{1}$$

where parameter a means the difference between the coordinate *Yt* in case of t equal 0 and the asymptote *K*, the parameter *b* characterizes the ratio of successive increases of the ordinate.

The exponent is modified to describe the process, influenced by a singular restricting factor, which influence is increasing in accordance with *Yt* growth. Although the restricting factor influences the process only in case of its exponential development till particular moment, then this process is mainly approximated by an S-curve with a point of inflection at which accelerated growth is slowed down. The modified exponent serves as a base curve, on which basis the Gomperz function and the logistic curve (such curves are used more often) are obtained through certain transformations.

As a rule, *the Pearl and Reid logistic curve* is presented as:

$$Y_t = \frac{K}{1 + be^{-at}}. \tag{2}$$

If the process indicator (proportion) varies from 0 to 1, then the formula of the logistic function is simplified:

$$Y_t = \frac{1}{e^{a+bt} + 1} \ or \ \frac{1}{Y_t} = 1 + e^{a+bt}, \tag{3}$$

where *e* is the basis of the natural logarithm; *a*, *b* – curve parameters.

At the same time, *Gompertz function* is presented in the following way:

$$Y_t = Ka^{b^t}. \tag{4}$$

The Pearl and Reid logistic curve and the Gompertz function describe the dynamic processes that concern the ratio of increments to ordinates change. Their differences are described in accordance with some peculiarities. In particular, Gompertz function has the ratio of the primary logarithm differences to the ordinates spaced apart and the Pearl and Reid logistic curve has the ratio of the first inverse ordinal differences. At the same time, the logistic curve is centrally symmetric to the inflection point. The Gompertz curve is asymmetric.

If the asymptote of one of the above-mentioned curves is unknown or cannot be determined, then the estimation of the function parameters is complicated. In these cases, different methods of analysis can be applied, namely: the three-sum method, the three-point method [21, p. 114-122], regression [21, p. 125-130], the Bryant method [22], Fisher, Yule, Rhodes, Neuro methods, etc.

The asymptote for the modified exponent is defined using the three-point method according to the following algorithm:

1. Data division into 3 groups I, II and III under the following conditions:

a) if the total number of elements is divisible by 3 without rest, namely $n = 3k$, then we form 3 groups, containing k elements for each one;

b) if $n = 3k + 1$, then group II consists of $k + 1$ elements, and groups I and III contain k elements for each one;

c) if $n = 3k + 2$, then group II consists of k elements and groups I and III - $k + 1$ elements each one.

2. The median value calculation within three groups. These values are marked respectively: y_{III}, y_{II}, y_I.

3. A system of three equations (nonlinear) with three unknowns is solved:

$$\begin{cases} y_I = \alpha\beta^{xI} + \gamma, \\ y_{II} = \alpha\beta^{xII} + \gamma, \\ y_{III} = \alpha\beta^{xIII} + \gamma. \end{cases}$$

This system can be solved as follows:

a) determine the difference between the second and first equation and between the third and second ones:

$$y_{III} - y_{II} = \alpha(\beta^{xIII} - \beta^{xII}), \tag{5}$$

$$y_{II} - y_I = \alpha(\beta^{xII} - \beta^{xI}), \tag{6}$$

b) divide term by term equation (5) by equation (6), marking:

$$\Delta = x_{III} - x_{II} = x_{II} - x_I;$$

$$\frac{y_{III} - y_{II}}{y_{II} - y_I} = \beta^{\Delta},$$

where: $\ln\beta = \dfrac{1}{\Delta}\ln\left(\dfrac{y_{III} - y_{II}}{y_{II} - y_I}\right).$

c) define: $\alpha = \dfrac{y_{III} - y_{II}}{\beta^{xIII} - \beta^{xII}}$ and $\gamma = y_I - \alpha\beta^{xI}$.

Finally, we get the following formula:

$$A = \frac{1}{\Delta}\ln\left(\frac{y_{III} - y_{II}}{y_{II} - y_I}\right), \beta = e^{A},$$

$$\alpha = \frac{y_{III} - y_{II}}{\beta^{xIII} - \beta^{xII}}, \gamma = y_I - \alpha\beta^{xI}.$$

4.2. Calculation of the reverse point (LPI) for the Ukrainian financial system

The data on GDP growth rate and the level of capital investment public financing in Ukrainian GDP in 2008-2018 are selected within the empirical research. The asymptote is determined, using the three-point method (the above-mentioned algorithm). In our case, the asymptote level is 7.44% (Table 1).

Table 1 The asymptote calculation in accordance with the three-point method

	GDP growth rate, %	**The public investment level within the economy, % to GDP**	**Normalized data**		**Xi**	**Yi**	**Δ**	**ln β**	**β**	**α**
I	32,93%	3,25%	1,31	1,98	0,67063	0,60904	-1,3893	7,44%	0,0773	- 6,003
	31,91%	2,82%	1,23	1,43						
	-4,42%	1,51%	- 1,75	- 0,21						
	18,32%	1,42%	0,11	- 0,32						
II	20,40%	1,86%	0,28	0,23	-0,71865	0,032176				
	8,15%	1,70%	- 0,72	0,03						
	4,36%	0,85%	- 1,03	- 1,04						
III	4,22%	0,55%	- 1,04	- 1,42	0,45645	-0,48801				
	25,31%	1,07%	0,69	- 0,77						
	19,96%	1,51%	0,25	- 0,21						
	25,05%	1,91%	0,66	0,29						
An average value	16,93%	1,68%								
σ	12,22%	0,80%								

The parameters of these curves are gradually determined, starting with the modified exponent. The function is considered by logarithm to a linear form to estimate *a* and *b* (Table 2).
Then, a system of normal equations of the form is constructed, using the least squares method:

$$\begin{cases}\sum \ln Y = n\ln a + \ln b\sum t,\\ \sum t\ln Y = \ln a\sum t + \ln b\sum t^2.\end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases}-31{,}486 = 11\ln a + 1{,}862\ln b\\ -5{,}454 = 1{,}862\ln a + 0{,}464\ln b\end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases}\ln a = -2{,}7217,\\ \ln b = -0{,}8310.\end{cases}$$

Table 2 Calculation of the modified exponent parameters

GDP growth rate	The public investment level within the economy, share of GDP	Y=У-limK	lnY	X	XlnY	X^2
X	**У**					
0,329	0,033	0,042	- 3,173	0,329	- 1,045	0,108
0,319	0,028	0,046	- 3,073	0,319	- 0,981	0,102
- 0,044	0,015	0,059	- 2,824	- 0,044	0,125	0,002
0,183	0,014	0,060	- 2,810	0,183	- 0,515	0,034
0,204	0,019	0,056	- 2,886	0,204	- 0,589	0,042
0,081	0,017	0,057	- 2,857	0,081	- 0,233	0,007
0,044	0,009	0,066	- 2,719	0,044	- 0,118	0,002
0,042	0,005	0,069	- 2,674	0,042	- 0,113	0,002
0,253	0,011	0,064	- 2,752	0,253	- 0,697	0,064
0,200	0,015	0,059	- 2,825	0,200	- 0,564	0,040
0,251	0,019	0,055	- 2,894	0,251	- 0,725	0,063
Σ	**0,184**	**-**	**- 31,486**	**1,862**	**- 5,454**	**0,464**

In case of the solution potentiation, the following parameters are obtained: a = *0,0658* and b = *0,4356*. As the result, we obtain the following equation: $Y=0{,}0658\times0{,}4356^t$. In case of moving from Y to the original equation of the modified exponent, we are getting the following expression: $\underline{Y_t=0{,}0744-0{,}0658\times0{,}4356^t}$.
Then, the Pearl-Reid curve is constructed. Although the authors have previously calculated the asymptote for all considered functions (Table 3), Fisher's method is additionally used for the data comparison within the logistic curve construction. The method is based on the determination of asymptote derivative; which t differentiation enables to provide the equation:

$$\frac{dy_t}{dt} = ay_t\left(1-\frac{y_t}{K}\right). \qquad (7)$$

Table 3. Parameters of the logistic curve calculation

GDP growth rate	The level of capital investment public financing, share of GDP	$z_t = \frac{1}{2}\ln\frac{Y_{t+1}}{Y_{t-1}}$	$Y = \left\lvert\frac{lim}{Y_t}-1\right\rvert$	*ln Y*
t	***Yt***			
0,329	0,033	-	-	-
0,319	0,028	-0,385	0,622	-0,474
-0,044	0,015	-0,341	0,294	-1,223
0,183	0,014	0,106	0,253	-1,375
0,204	0,019	0,089	0,429	-0,847
0,081	0,017	-0,391	0,375	-0,980
0,044	0,009	-0,569	0,249	-1,391
0,042	0,005	0,112	0,950	-0,051
0,253	0,011	0,510	0,001	-6,736
0,200	0,015	0,291	0,297	-1,215
0,251	0,019	-	0,442	-0,817
Σ	**0,184**	**-**	**-**	**-15,1102**

The growth rate is marked through z_t, where $z_t = a - \frac{a}{K} y_t$. Namely, there is a possibility to assume that the intervals between the levels of a series of dynamics can be equal and the z_t estimation may be carried out in the form of the following equation by prolinearizing expression (7):

$z_t = \frac{1}{2} \ln \frac{y_{t+1}}{y_{t-1}}$, where *t = 2, 3,…, n-1*.

Then, the least squares method is used to construct the regression equation, which in our case has the following form: $Z_t=0,1657–15,571y_t \rightarrow a=0,1657$.

In case $-\frac{a}{K} = -15,5751$,

then $K = \frac{-0,1657}{-15,5751} = 0,01064$, the upper asymptote of public investment is approximately 1.06%.

Primarily, the parameters *a* and *K* are provided. Then, the parameter *b* is calculated. The function $Y_t = \frac{K}{1+be^{-at}}$ is represented as $\frac{K}{y_t} - 1 = be^{-at}$ for these goals. The equation $Y = be^{-at} \rightarrow \ln Y = \ln b + at$ is obtained through denoting the left part of equality by *Y* and providing its logarithm. In the above-mentioned expression, *b* is a free term that can be found from the first equation of the system of normal equations module in the following way:

$$|\ln b| = \frac{1}{11} \cdot (-15,1102) + 0,1657 \cdot 0,1693 = \\ = 1,3456 \rightarrow b = 0,2968. \quad (8)$$

Thereby, the logistic curve can be represented in the following way: $Y_t = \frac{0,01064}{1+0,2968e^{0,1657t}}$.

The Gompertz function $Y_t = Ka^{b^t}$ is chosen as the third mathematical function to predict and model the processes of capital investments public financing. Its parameters can also be estimated by the least squares method. If the asymptote is given, then this function is reduced to a linear look by double logarithm:

$$\lg\left(\frac{y_t}{K}\right) = b^t \lg a \rightarrow \lg\left(\lg\left(\frac{y_t}{K}\right)\right) = t \lg b + \lg(\lg a) \quad (9)$$

After replacing $\lg\left(\lg\left(\frac{y_t}{K}\right)\right)$ through y', $\lg b$ through B and $\lg(\lg a)$ through A, the Gomperz function is represented in linear form: $y' = A + Bt$.

In the case of unknown asymptote, it may be determined using the regression method in the same way as for the modified exponent. For this purpose, it is necessary to transform the Gomperz function into a modified exponent by prologarifying the equation:

$$\ln y = \ln K + \ln a \cdot b^t \rightarrow Y = k + A \cdot b^t,$$

where $Y = \ln y; k = \ln K; A = \ln a$. Then, the authors find the absolute increments and express them through the parameters of the modified exponent:

$$\Delta Y = k + Ab^t - k - Ab^{t-1} = Ab^{t-1} \cdot (b-1).$$

Recalculating this equality through logarithm, we get:

$$\ln \Delta Y = \ln A + (t-1)\ln b + \ln(b-1).$$

Expressing $\ln A + \ln(b-1)$ through d, the authors find the linear equation in logarithms $\ln \Delta Y = d + (t-1)\ln b$, the parameters can already be estimated through the least squares method. The calculation of the parameters for the Gomperz function creation is presented in Table 4.

Table 4 Calculation of the Gomperz function parameters

GDP growth rate	The level of capital investment public financing, share of GDP	ln Y	|Δ ln Y|	ln (Δ ln Y)	t-1	k
t	*Y*					
0,329	0,033	-3,425	-	-	-0,671	-3,410
0,319	0,028	-3,570	0,145	-1,932	-0,681	-3,555
-0,044	0,015	-4,195	0,625	-0,470	-1,044	-4,157
0,183	0,014	-4,252	0,057	-2,863	-0,817	-4,231
0,204	0,019	-3,983	0,268	-1,315	-0,796	-3,963
0,081	0,017	-4,073	0,090	-2,413	-0,919	-4,045
0,044	0,009	-4,765	0,692	-0,367	-0,956	-4,735
0,042	0,005	-5,211	0,446	-0,808	-0,958	-5,181
0,253	0,011	-4,542	0,669	-0,402	-0,747	-4,524
0,200	0,015	-4,191	0,351	-1,048	-0,800	-4,171
0,251	0,019	-3,960	0,231	-1,464	-0,749	-3,942

As a result, the equation is obtained: $\ln \Delta Y = -3,4623 - 2,5438(t-1)$.

Then, $b = e^{-2,5438} = 0,0786$, and $d = -3,4623$.

At the same time $\ln a \cdot (b-1) = e^{-3,4623} = 0,0314$;

$$\ln a = \frac{0{,}0314}{(0{,}0786-1)} = -0{,}0340;\ a = 0{,}9665.$$

The parameters a and b are distinguished. Thereby, authors can find k for each entry from Table 4 as $k = Y - Ab^t$. Then, an average value of k is estimated. The upper asymptote $k = \ln K \rightarrow K = e^k$ is determined on its basis. In our case $K = e^{-4{,}174} = 0{,}015$. After paying attention to these calculations, the Gompertz function is presented in the following way: $Y_t = 0{,}015 \cdot 0{,}9665^{0{,}0786^t}$.

Each of the above-mentioned features will be tested against a number of criteria to identify the best fit for the selected model. The following criteria are selected:

1) Adjusted Multiple Determination Ratio:

$$\bar{R}^2 = 1 - \frac{\dfrac{\sum_{i=1}^{n}\varepsilon_i^2}{n-m-1}}{\dfrac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n-1}} = 1 - \frac{\dfrac{D(\varepsilon)}{n-m-1}}{\dfrac{D(y)}{n-1}} = 1 - \frac{s_\varepsilon^2}{s_y^2}, \tag{10}$$

where *D(ε)* – selective residual variance, *D(y)* – selective variance of the dependent variable, *s2ε*– the unbiased estimate of the variance of the residuals, *s2y*– the unbiased estimation of the variance of the dependent variable. The model will be better in case of the greater the value of the adjusted multiple determination factor.

2) Akaike's Information Criterion:

$$AIC = \lg\left(\frac{1}{n}\sum_{i=1}^{n}\varepsilon_i^2\right) + 2\cdot\frac{m+1}{n}. \tag{11}$$

In accordance with this criteria, more specified model has lower AIC value.

3) Bayesian Information Criterion:

$$BIC = \lg\left(\frac{1}{n}\sum_{i=1}^{n}\varepsilon_i^2\right) + \frac{m+1}{n}\cdot\log n. \tag{12}$$

In accordance with this criteria, the model is better approximated in case of smaller BIC value.

The results of the calculation of the above-mentioned criteria are represented in Table. 5.

Table 5 Comparative table of functions selection criteria

Function	R^2	$\bar{R}^2$	AIC	BIC
Modified exponent	0,4773	0,4192	-1,6252	-1,5528
Perl and Reid logistic curve	0,0769	-0,055	-0,5109	-0,4671
Gompertz function	0,1155	0,0049	0,1779	0,2384

The modified exponent showed the most adequate results in the verification of the criteria fulfillment. Thereby, the further conclusions are provided on its basis.

5. CONCLUSION

The quantitative manifestation of the government direct participation in the investment processes is the amount of the budget capital expenditures, their share in total budget expenditures and the correlation of government capital expenditures to the GDP of the country. Public investment, on the one hand, contributes to economic growth and, on the other hand, encourages this growth by reducing the motivation of market agents.

It is proposed to determine the threshold value of the state funding for capital investments by constructing S-curves: a modified exponent, the Pearl-Reid logistic curve and the Gompertz curve. According to the results of empirical studies in Ukraine, the Modified exponent showed the most adequate results in the verification of the fulfillment of the criteria (adjusted multiple coefficient, Akaike's, Schwartz).

Application of the mentioned above methodology enabled us to make the following conclusions: if government spending in the GDP structure accounts for 33-35%, the size of public investment in Ukraine should be within 7% of GDP (LPI), meanwhile the growth of the country's economy in the form of GDP growth will not be possible when the state reduces capital investments to 0.86%.

Comparing the reversal point with the real data in Ukraine, it is not difficult to notice that the amount of public financing of the capital investment is not sufficient and has a random and irregular nature of their realization. Thus, according to the State Statistics Service of Ukraine [23] data during 2008-2018, the level of capital investments public financing was the lowest one in 2014 (0,55% of GDP) and the highest one in 2008 (2,8% of GDP), it was 2.06% of GDP as of the beginning of 2019.

The Ukrainian government limits the investment function of the country's main financial plan by creating a target for overcoming or minimizing budget deficits in the social focus of budget resources. Finally, this leads to inefficient use of budget levers, lack of financing and imbalances within entire industries and regions, slowing down the economy and innovative development.

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